Obstacle-Avoidance Verification for a Switched Control Strategy

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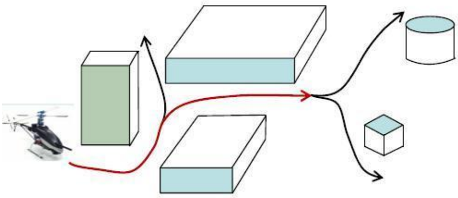
December 16, 2012

Outline

Motivation

A Motivating Example



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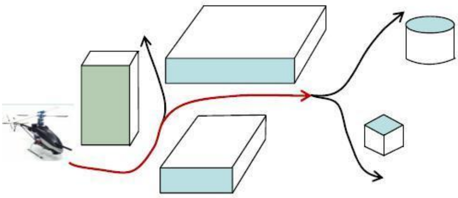

- Trajectory known up to a finite future horizon

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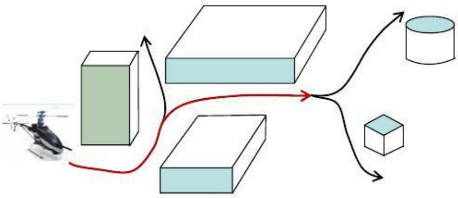

- Trajectory known up to a finite future horizon
- *Set* of possible future trajectories known.

Hybrid System Model

- Model behavior as a hybrid system
    - Continuous-state dynamics
    - Discrete switching in plant parameters

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- Can we guarantee trajectory tracking?
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- Can we guarantee collision avoidance?
  - Analysis using SpaceEx

Control of Switched Systems

Switched linear systems

- A collection of (linear) plant parameters

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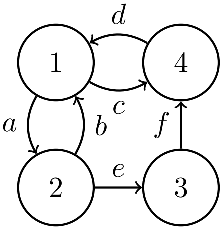
Switched linear systems



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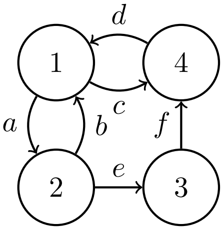

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- Discrete switching logic selects the parameters at each time
- Switching graph is known, but exact switching sequence is not
- System dynamics given by

$$\begin{aligned} x_{t+1} &= A_{\theta(t)} x_t + B_{\theta(t)} w_t \\ z_t &= C_{\theta(t)} x_t + D_{\theta(t)} w_t \end{aligned}$$

Finite-path controllers

- Controller has access to plant output and switching signal

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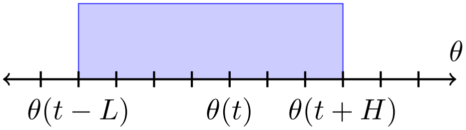
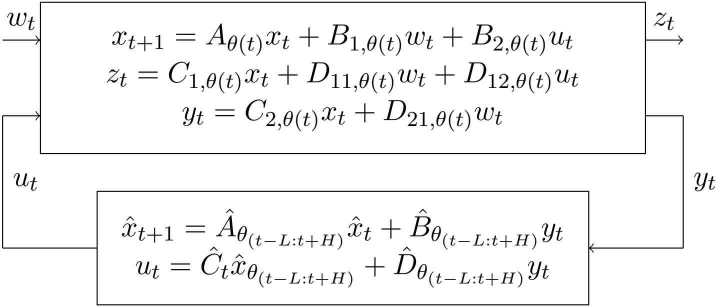
- Perfect observation/memory of current, past modes
- Preview of a finite-horizon of future modes
- Controller parameters depend on this switching path



ℓ_2-induced-norm performance



$$
\begin{aligned}
x_{t+1} &= A_{\theta(t)} x_t + B_{1,\theta(t)} w_t + B_{2,\theta(t)} u_t \\
z_t &= C_{1,\theta(t)} x_t + D_{11,\theta(t)} w_t + D_{12,\theta(t)} u_t \\
y_t &= C_{2,\theta(t)} x_t + D_{21,\theta(t)} w_t
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$$

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\hat{x}_{t+1} &= \hat{A}_{\theta_{(t-L:t+H)}} \hat{x}_t + \hat{B}_{\theta_{(t-L:t+H)}} y_t \\
u_t &= \hat{C}_t \hat{x}_{\theta_{(t-L:t+H)}} + \hat{D}_{\theta_{(t-L:t+H)}} y_t
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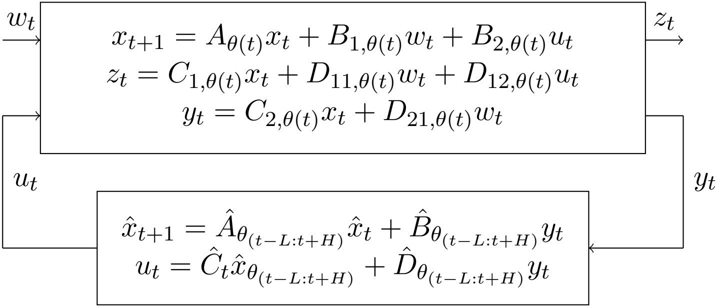

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Is there a stabilizing controller which bounds the system norm $w \mapsto z$ uniformly?

Brief summary of proof strategy

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- Find a (finite) collection of Lyapunov functions
- Arrange them in the correct order for each possible switching sequence
- Size of collection is dependent on the length of the switching window
- Conditions are both necessary and sufficient

Existence conditions for a controller

There exists a controller with $L \geq 0$ and $H \geq 0$ achieving attenuation level γ if and only if there exist an integer $M \geq 0$ and matrices $R_j \succ 0$, $S_j \succ 0$ such that

$$N_{F,i_0}^T \begin{bmatrix} A_{i_0} R_{i_-} A_{i_0}^T - R_{i_+} & A_{i_0} R_{i_-} C_{1,i_0}^T & B_{1,i_0} \\ C_{1,i_0} R_{i_-} A_{i_0}^T & C_{1,i_0} R_{i_-} C_{1,i_0}^T - \gamma I & D_{11,i_0} \\ B_{1,i_0}^T & D_{11,i_0}^T & -\gamma \end{bmatrix} N_{F,i_0} \prec 0$$

$$N_{G,i_0}^T \begin{bmatrix} A_{i_0}^T S_{i_+} A_{i_0} - S_{i_-} & A_{i_0}^T S_{i_+} B_{1,i_0} & C_{1,i_0}^T \\ B_{1,i_0}^T S_{i_+} A_{i_0} & B_{1,i_0}^T S_{i_+} B_{1,i_0} - \gamma I & D_{11,i_0}^T \\ C_{1,i_0} & D_{11,i_0} & -\gamma I \end{bmatrix} N_{G,i_0} \prec 0$$

$$\begin{bmatrix} R_{i_-} & I \\ I & S_{i_-} \end{bmatrix} \succeq 0$$

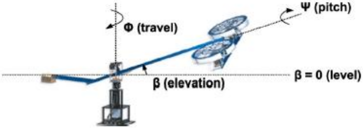
for all admissible sequences $i_{-L-M:H}$, where

$i_- = i_{(-L-M:H-1)}$, $i_+ = i_{(-L-M+1:H)}$ and

$$N_{F,i} = \begin{bmatrix} N(\begin{bmatrix} B_{2,i}^T & D_{12,i}^T \end{bmatrix}) & 0 \\ 0 & I \end{bmatrix}, \quad N_{G,i} = \begin{bmatrix} N([C_{2,i} \quad D_{21,i}]) & 0 \\ 0 & I \end{bmatrix}$$

3DOF Modeling

3DOF helicopter system



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[1] C.W. Dever, "Parameterized maneuvers for autonomous vehicles," Ph.D. dissertation, Dept. Mech. Eng., Massachusetts Institute of Technology, Cambridge, 2004

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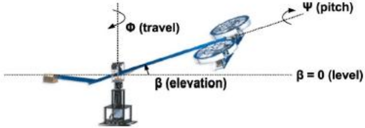

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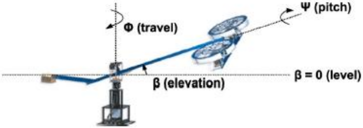

- Tabletop mounted system from Quanser Consulting
- Nonlinear dynamics[1] are given by

$$\ddot{\phi} = -0.0252\dot{\phi} - 0.0525V_c^2\sin(\psi - 0.0827)$$

$$\ddot{\beta} = -0.112\dot{\beta} - 0.243\beta - 0.504\sin\beta + 0.04\dot{\phi}^2 + 0.0905V_c^2\cos\psi$$

$$\ddot{\psi} = -0.163\dot{\psi} - 1.58\sin\psi + 0.131 - 0.449\dot{\phi}^2 + 1.42V_cV_y$$

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Reference trajectory and linearization

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- Helicopter will travel along $\dot{\phi}_r = -1$ rad/s and $\beta_r = 0.2618$ rad
- Modification of hover dynamics (non-zero $\dot{\phi}_r$) with disturbance
- Resulting system:

$$\begin{aligned}
\ddot{\phi} &= -.257\psi - 0.0839\dot{\phi} + w_1 \\
\ddot{\beta} &= -.504\beta - .112\dot{\beta} + 1.34\tau_c + w_2 \\
\ddot{\psi} &= -1.58\psi - .163\dot{\psi} + 16.2\tau_y + w_3 \\
\dot{\tau}_c &= -6.16\tau_c + V_c \\
\dot{\tau}_y &= -7.32\tau_y + V_y
\end{aligned}$$

Critical outputs

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- For reference tracking, introduce the integral error ξ such that $\dot{\xi} = \zeta - \zeta_r$
- When near an obstacle, ζ and ξ represent "critical" outputs.

Weighting of critical outputs

- Trade-off between altitude (ζ,ξ) and travel $(\phi,\dot{\phi})$

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- Let $\delta \in [0,1]$ be a "danger" parameter
- Controlled output given by

$$z = \begin{bmatrix} (1-.9\delta)(\phi+0.5\dot{\phi}) \\ (1+.9\delta)(\zeta+0.1\xi) \\ .25V_c \\ .25V_y \end{bmatrix}$$

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- Allow δ to switch between adjacent values, or to remain at either 0 or 1.

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- Solving the existence conditions for this system produces a suitable modal controller.
- Path-dependent controllers are also possible
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  - Number of controller modes grows quickly
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SpaceEx Verification

Reachability and collision avoidance

- Does the altitude error ever grow large enough to cause a collision?
    - What is the reachable set of plant states?

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- Determine bounds on critical outputs

Computational Difficulties

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- Bounds can be placed on w at each time, but not on total signal norm
- Result: Bounds on reachable states are insufficient to guarantee collision avoidance

Results and future work

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- Possible solutions:
  - Improved hardware - parallel algorithms for efficient search.
  - Formal verification - find worst-case switching logic/disturbance

Questions?

Thank you!