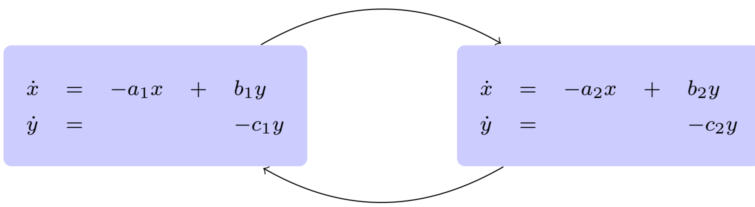
ECE/CS 584: Embedded System Verification Fall 2012

Homework 3— Due on Dec 4^{th}, 2012 *Your name*

Problem 1 (20 points). In the following hybrid system $\mathcal{A}$, $a_1, a_2, c_1\mathbb{R}_+$ and $b_1, b_2 \in \mathbb{R}$.



(a) Assuming, $c_2 \in \mathbb{R}_+$ show that $\mathcal{A}$ is globally exponentially stable under arbitrary switching.

(b) Assuming some restrictions on the switching signal, show that $\mathcal{A}$ is globally exponentially stable (for arbitrary c_2).

Problem 2 (20 points). (a) Convert the following rectangular initialized hybrid automaton to a timed automaton[1]. The first expression on the arrows are the preconditions/guards and the second expresssion is the effect or the reset function.

(b) Plot an execution of the original hybrid automaton and the corresponding execution of the timed automaton.

Problem 3 (20 points). Consider a system with two leaky tanks T_1 and T_2 and an inflow pipe P which can feed to either of the tanks. The inflow rate from P, when on, is f_{in}, and the outflow rates from the tanks (independent of any inflow) are f_1 and f_2. These rates are measured in terms of the rate of drop (and rise) of the water levels in the tanks. The controller for P is designed such that within δ time of the level in tank i dropping below h_i, the pipe P is turned to feed T_i.

(a) Model the system as a hybrid automaton. Show the circle-arrow representation.

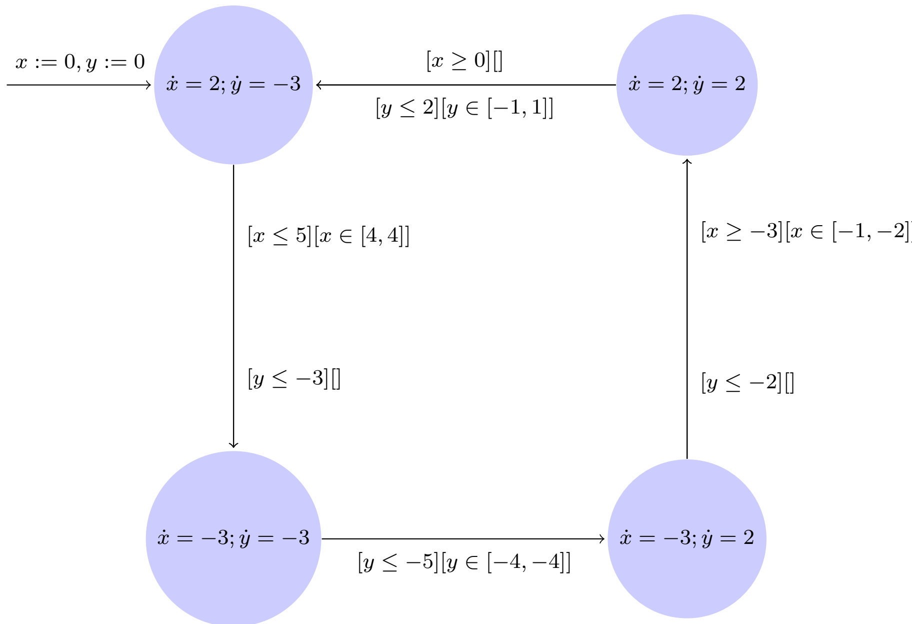
(b) Under what conditions does the model display Zeno behavior ?

(c) Under what conditions can it be guaranteed that neither tank becomes empty? Prove it.

Problem 4 (20 points) Let $C[0,1]$ be the set of continuous functions from $[0,1] \to \mathbb{R}$. Define $\sqsubseteq$ on $C[0,1]$ by $f \sqsubseteq g$ if and only if for all $a \in [0,1]$, $f(a) \leq g(a)$.

(a) Show that $\sqsubseteq$ is a partial order and that $\langle C[0,1], \sqsubseteq \rangle$ forms a lattice.

[1] clocks may be initialized to constant intervals.



(b) Does an analogous statement hold if we consider the set of differentiable functions from $[0, 1] \to \mathbb{R}$?

Problem 5 (20 points) Suppose $\langle L, \alpha, \gamma, M \rangle$ is a Galois connection. That is $\alpha : L \to M$, $\gamma : M \to L$, both are monotonic, and $\alpha \circ \gamma \sqsubseteq_M \lambda m.m$ and $\gamma \circ \alpha \sqsupseteq_L \lambda l.l$.

(a) Show that the above conditions are equivalent to the condition that for any $l \in L, m \in M$, $\alpha(l) \sqsubseteq_M m \iff l \sqsubseteq_L \gamma(m)$.

(b) Show that condition (a) is equivalent to the conjunction of (i) $\forall\, l \in L$, $l \sqsubseteq_L \gamma(\alpha(l))$ and (ii) $\forall\, m \in M$, $\alpha(\gamma(m)) \sqsubseteq_M m$.

(c) Suppose L is the set of sets of concrete states of some hybrid automaton and M is an abstract domain for analysis. In that case, what makes more sense: $l = \gamma(\alpha(l))$ or $m = \alpha(\gamma(m))$, and why?