PVS Tutorial (Part 1 & 2)

ECE/CS 584: lecture 06 & 07

sayan mitra

mitras@illinois.edu

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

September 20 & 25, 2012

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  - + expressive
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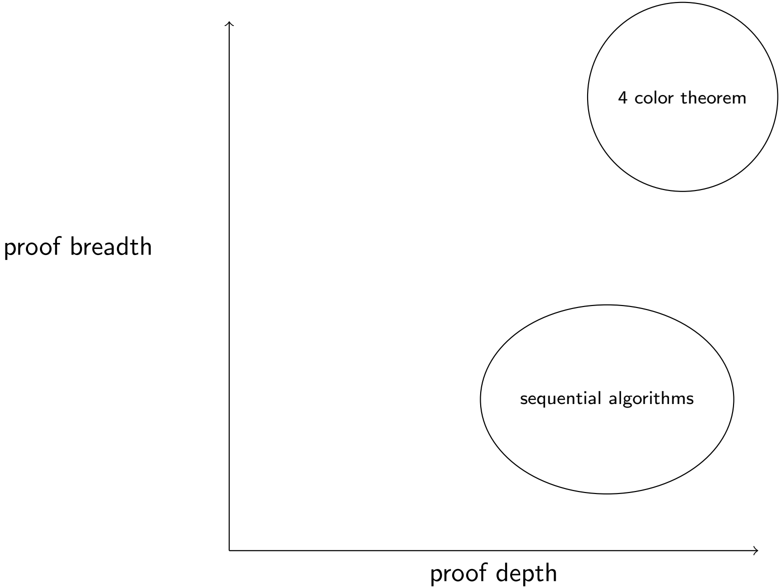
  - + automatically check proof after changing specs
  - successful in large critical systems, e.g., NASA, JPL, Transportation system
  - - not automatic in general
  - - requires expertise

current theorem prover technology



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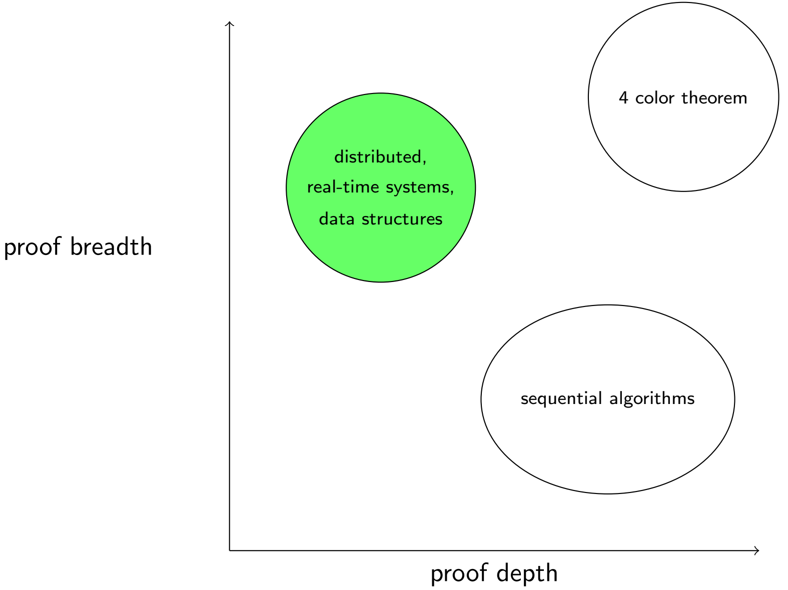

overview of tutorial

- quick introduction to PVS—a theorem prover for high-order logic
  - PVS specification language
  - prover commands
- specifying hybrid/real-time/distributed systems (HIOA) in PVS
- proving properties of using PVS

propositional logic

$$P := \mathit{true} \mid \mathit{false} \mid \neg P_1 \mid P_1 \wedge P_2 \mid P_1 \vee P_2 \mid P_1 \implies P_2 \mid P_1 \iff P_2$$

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many interesting problems can be expressed in propositional logic, e.g., circuit design, hardware verification

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  - harder to decide $\Rightarrow$ fully automatic verification not possible

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- PVS system guide
  `http://pvs.csl.sri.com/doc/pvs-system-guide.pdf`
  Read chapter 2 for basic instructions about the user interface
- PVS language
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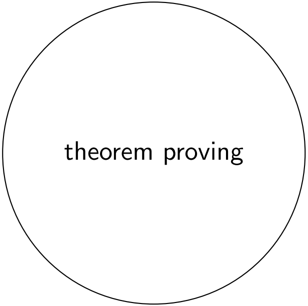
theorem proving and other areas



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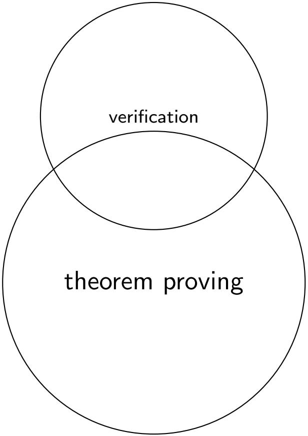

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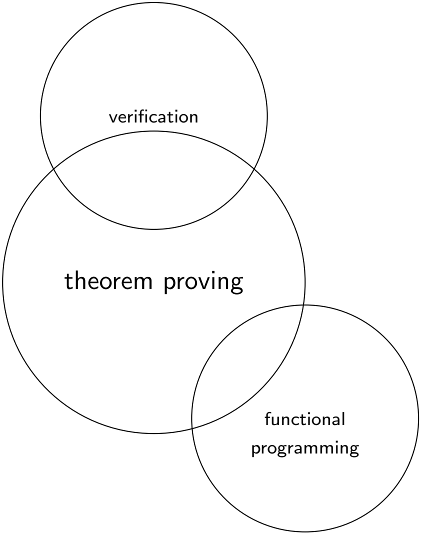

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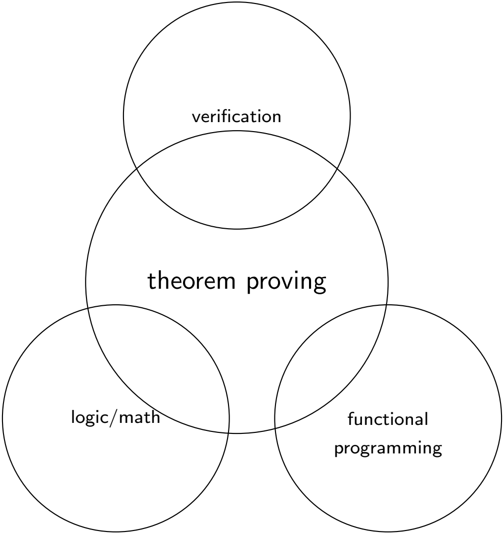

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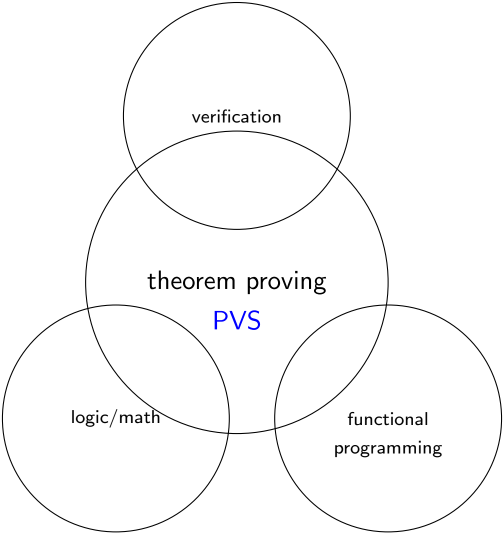

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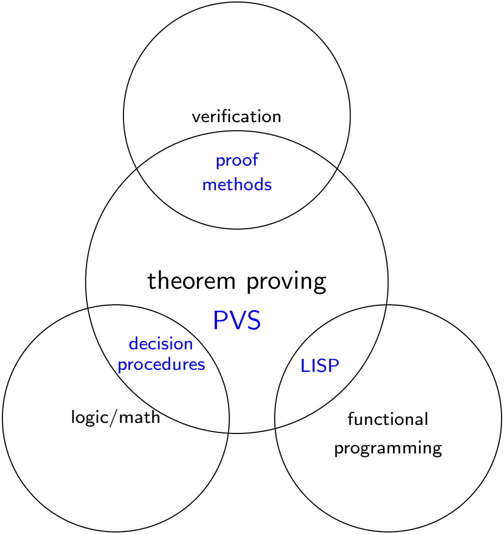

example 1: a theory of stack of integers

Stack: **theory begin**

Stack: **type** = [# *length*: **nat**, *seq*: [*below*[*length*] -> **nat**] #]

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top(*c*:*NonEmptyStack*):**nat** = *q*‘*seq*(*length*(*c*)-1)

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top(c:NonEmptyStack):nat = q`seq(length(c)-1)

push(c:stack, a:nat):NonEmptyStack =
(# length := c`length + 1,
seq := seq(c) with [(c`length) := a] #)

pop(c:NonEmptyStack):[Stack,nat]

end Stack
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- all assignments and definitions must be type-correct
- typechecking is in general undecidable; PVS generates proof obligations or type correctness conditions (TCCs). E.g., application of *pop*(c) generates the TCC *NonEmptyStack?*(c)

some properties of stacks

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. . .
c: var Stack
a: var nat

nonempty: lemma forall (c,a): NonEmptyStack?(push(c,a))

idem : lemma forall (c, a): pop(push(c , a))`1 = c

pushpop: lemma forall (c, a): pop(push(c,a))`2 = a

end Stack
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a polymorphic stack

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Stack[T:type+]: theory begin

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inductive definitions and recursive functions

$\mathit{even}(n{:}\textbf{nat})$: **inductive bool** $= n = \mathbf{0}$ **or** $n > \mathbf{1}$ **and** $\mathit{even}(n\text{-}\mathbf{2})$

inductive definitions and recursive functions

even(*n*:**nat**): **inductive bool** = *n* = **0 or** *n* > **1 and** *even*(*n*-**2**)

fact(*n*:**nat**): **recursive nat** = **if** *n* = **0 then 1 else** *n* ∗ *fact*(*n*-**1**) **endif**
measure lambda (*n*:**nat**):*n*

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- mutual recursion not allowed
- domain of the measure function is the same domain as the recursive function being defined and its range must be a well-founded set with a order relation

polymorphic theory of automata

simplemachine[
states, *actions*: **type**,
enabled: [*actions*,*states* -> **bool**],
trans: [*actions*,*states* -> *states*],
start: [*states* -> **bool**]
]: **theory**

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 reachable_hidden(s,n): recursive bool =
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enabled(a,s1) and s = trans(a,s1))
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reachable(s): bool = exists n : reachable_hidden(s,n)
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inductthm(*Inv*): **bool** = *base*(*Inv*) **and** *inductstep*(*Inv*)
implies (**forall** *s* : *reachable*(*s*) **implies** *Inv*(*s*))

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no! push internal nondeterministic choices to (external) choice over actions

many more types of types

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ID:**type** = {**1**,**2**,**3**,**4**}
location:**type** = [*x*:**real**, *y*:**real**]

states: [# *pos*:[*ID* -> *location*], *clock*:[*ID* -> **posreal**], *failed*:[*ID* -> **bool**] #]

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actions: **datatype**
fail(*i*:*ID*):*fail*?
time_elapse(*t*:**posreal**):*time_elapse*?
send(*i*:*ID*,*m*:*location*):*send*?
receive(*i*:*ID*,*m*:*location*):*receive*?
end *actions*

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- defines a new type called actions

abstract datatypes

an abstract datatype defines a collection of objects through constructors and recognizers.

actions: **datatype**
fail(*i*:*ID*):*fail*?
time_elapse(*t*:**posreal**):*time_elapse*?
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  - *time_elapse*?(*a_f3*) returns false
  - *i*(*a_f3*) returns 3
  - what is *i*(*time_elapse*(**10**)) ?

defining enabling conditions and transitions

enabled(*a*:*actions*, *s*:*states*):**bool** =
cases *a* **of**
fail(*i*):
not *failed*(*s*)(*i*)

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send(*i*,*m*):
pos(*s*)(*i*) = *m*
...
endcases

defining enabling conditions and transitions

```
enabled(a:actions, s:states):bool =
cases a of
fail(i):
not failed(s)(i)

send(i,m):
pos(s)(i) = m
...
endcases

trans(a:actions, s:states):states =
 cases a of
time_elapse(t):
s with [clock :=  clock(s) + t]
```

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trans(a:actions, s:states):states =
cases a of
time_elapse(t):
s with [clock := clock(s) + t]

fail(i):
s with [failed := failed(s) with [(i) := true]
...
endcases
```

review of language constructs

- theory: a collection of type and function definitions, axioms, and theorems

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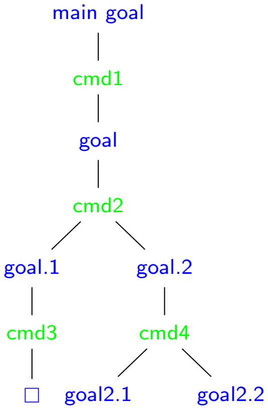
- a predicate on type T automatically defines a subtype of T, e.g., *NonEmptyStack*? is a subtype of *Stack*
- all assignments and definitions must be type-correct
- typechecking is in general undecidable; PVS generates proof obligations or type correctness conditions (TCCs). E.g., application of *pop*(*c*) generates the TCC *NonEmptyStack*?(*c*)

PVS prover

- user interacts with PVS to construct a proof tree
- each node of the tree is a proof goal
- parent goal follows from the children by means of a proof step



proof goals and sequents

a proof goal is a sequent a sequence of formulas

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a sequent S is represented as represented as

{-**1**} *A1*
{-**2**} *A2*
[-**3**] *A3*
...
$\vdash$ – –
{-**1**} *B1*
[-**2**] *B2*
[-**3**] *B3*
....

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a proof goal is a sequent a sequence of formulas
a sequent S is represented as represented as

{-**1**} *A1*
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...
$\vdash$ – –
{-**1**} *B1*
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[-**3**] *B3*
....

A1, *A2*, *A3*, ... are called antecedents and *B1*, *B2*, *B3*, ... are consequents

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[-**2**] *B2*
[-**3**] *B3*
....

A1, *A2*, *A3*, ... are called antecedents and *B1*, *B2*, *B3*, ... are consequents
interpretation: $\text{A1} \wedge \text{A2} \wedge \text{A3} \wedge \ldots \implies \text{B1} \vee \text{B2} \vee \text{B3} \vee \ldots$

PVS prover commands

- primitive rules
  - propositional rules
  - quantifier rules
  - equality rules
  - structural rules
  - control rules
  - others: using lemmas, induction, extensionality, decision procedures

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- primitive rules
  - propositional rules
  - quantifier rules
  - equality rules
  - structural rules
  - control rules
  - others: using lemmas, induction, extensionality, decision procedures
- commands and keywords for combining primitive rules into strategies (not covered in this lecture)

propositional rules: flatten

performs disjunctive simplification

{-**1**} *A1*
{-**2**} **not** *A2*
$\vdash$ – –
{**1**} *B1*

Rule **?** (**flatten**)

propositional rules: flatten

performs disjunctive simplification

{-**1**} *A1*
{-**2**} **not** *A2*
⊢ – –
{**1**} *B1*

Rule **?** (**flatten**)

[-**1**] *A1*
⊢ – –
[**1**] *B1*
{**2**} *A2*

propositional rules: flatten

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{-**1**} *A1*
{-**2**} **not** *A2*
⊢ – –
{**1**} *B1*

Rule **?** (**flatten**)

[-**1**] *A1*
⊢ – –
[**1**] *B1*
{**2**} *A2*

[-**1**] *A1* **and** *A2*
⊢ – –
{**1**} *B1* **implies** *B2*

Rule **?** (**flatten**)

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{-**1**} *A1*
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⊢ – –
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Rule **?** (**flatten**)

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⊢ – –
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{**1**} *B1* **implies** *B2*

Rule **?** (**flatten**)

{-**1**} *A1*
{-**2**} *A2*
{-**3**} *B1*
⊢ – –
{**1**} *B2*

propositional rules: split

splits a conjunctive formula in the current goal and collects the resulting subgoal(s)

{-**1**} *A1*
$\vdash$ – –
{**1**} *B1* **and** *B2*

Rule **?** (**split** **1**)

propositional rules: split

splits a conjunctive formula in the current goal and collects the resulting subgoal(s)

{-**1**} *A1*
⊢ – –
{**1**} *B1* **and** *B2*

Rule **?** (**split** **1**)

Subgoal.**1**
[-**1**] *A1*
⊢ – –
{**1**} *B1*

Subgoal.**2**
[-**1**] *A1*
⊢ – –
{**1**} *B2*

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splits a conjunctive formula in the current goal and collects the resulting subgoal(s)

{-**1**} *A1*
⊢ — —
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Rule **?** (**split** **1**)

Subgoal.**1**
[-**1**] *A1*
⊢ — —
{**1**} *B1*

Subgoal.**2**
[-**1**] *A1*
⊢ — —
{**1**} *B2*

⊢ — —
[**1**] *A1 iff A2*

Rule **?** (**split**)

propositional rules: split

splits a conjunctive formula in the current goal and collects the resulting subgoal(s)

{-**1**} *A1*
⊢ — —
{**1**} *B1* **and** *B2*

Rule **?** (**split** **1**)

Subgoal.**1**
[-**1**] *A1*
⊢ — —
{**1**} *B1*

Subgoal.**2**
[-**1**] *A1*
⊢ — —
{**1**} *B2*

⊢ — —
[**1**] *A1 iff A2*

Rule **?** (**split**)

Subgoal.**1**
⊢ — —
{**1**} *A1* **implies** *A2*

propositional rules: split

splits a conjunctive formula in the current goal and collects the resulting subgoal(s)

{-**1**} *A1*
⊢ — —
{**1**} *B1* **and** *B2*

Rule **?** (**split** **1**)

Subgoal.**1**
[-**1**] *A1*
⊢ — —
{**1**} *B1*

Subgoal.**2**
[-**1**] *A1*
⊢ — —
{**1**} *B2*

⊢ — —
[**1**] *A1 iff A2*

Rule **?** (**split**)

Subgoal.**1**
⊢ — —
{**1**} *A1* **implies** *A2*

Subgoal.**2**
⊢ — —
{**1**} *A2* **implies** *A1*

propositional rules: lift-if

lifts branching structure to the top level

$\vdash$ – –

$\{\mathbf{1}\}$ *foo*(**IF**(A,B,C))

Rule **?** (*lift*-**if**)

propositional rules: lift-if

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$\vdash$ – –

$\{\mathbf{1}\}$ *foo*(**IF**(*A*,*B*,*C*))

Rule **?** (*lift*-**if**)

$\vdash$ – –

$[\mathbf{1}]$ **IF**(*A*, *foo*(*B*), *foo*(*C*))

Rule **?** (**split**)

propositional rules: lift-if

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$[\mathbf{1}]$ **IF**(*A*, *foo*(*B*), *foo*(*C*))

Rule **?** (**split**)

Subgoal.**1**

$\vdash$ – –

$\{\mathbf{1}\}$ *A* **implies** *foo*(*B*)

propositional rules: lift-if

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$\vdash - -$

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Rule **?** (*lift-***if**)

$\vdash - -$

$[\mathbf{1}]$ **IF**(*A*, *foo*(*B*), *foo*(*C*))

Rule **?** (**split**)

Subgoal.**1**

$\vdash - -$

$\{\mathbf{1}\}$ *A* **implies** *foo*(*B*)

Subgoal.**2**

$\vdash - -$

$\{\mathbf{1}\}$ **not** *A* **implies** *foo*(*C*)

propositional rules: lift-if

lifts branching structure to the top level

$\vdash --$

$\{\mathbf{1}\}$ *foo*(**IF**(A,B,C))

Rule **?** (*lift*-**if**)

$\vdash --$

$[\mathbf{1}]$ **IF**(A, *foo*(B), *foo*(C))

Rule **?** (**split**)

Subgoal.**1**

$\vdash --$

$\{\mathbf{1}\}$ A **implies** *foo*(B)

Subgoal.**2**

$\vdash --$

$\{\mathbf{1}\}$ **not** A **implies** *foo*(C)

Subgoal.**1**

$\{\text{-}\mathbf{1}\}$ A

$\vdash --$

$\{\mathbf{1}\}$ *foo*(B)

propositional rules: lift-if

lifts branching structure to the top level

$\vdash - -$

$\{\mathbf{1}\}$ *foo*(**IF**(*A*,*B*,*C*))

Rule **?** (*lift*-**if**)

$\vdash - -$

[**1**] **IF**(*A*, *foo*(*B*), *foo*(*C*))

Rule **?** (**split**)

Subgoal.**1**

$\vdash - -$

$\{\mathbf{1}\}$ *A* **implies** *foo*(*B*)

Subgoal.**2**

$\vdash - -$

$\{\mathbf{1}\}$ **not** *A* **implies** *foo*(*C*)

Subgoal.**1**

$\{\text{-}\mathbf{1}\}$ *A*

$\vdash - -$

$\{\mathbf{1}\}$ *foo*(*B*)

Subgoal.**2**

$\vdash - -$

$\{\mathbf{1}\}$ *A*

$\{\mathbf{2}\}$ *foo*(*C*)

propositional rules: case

splits current proof goal based on sequence of assumptions

[-**1**] *A*
$\vdash$ – –
{**1**} *B*

Rule **?** (**case** *C1 C2*)

propositional rules: case

splits current proof goal based on sequence of assumptions

[-**1**] *A*
$\vdash$ – –
{**1**} *B*

Rule **?** (**case** *C1 C2*)

Subgoal.**1**

{-**1**} *C2*
{-**2**} *C1*
[-**3**] *A*
$\vdash$ – –
[**1**] *B*

propositional rules: case

splits current proof goal based on sequence of assumptions

[-**1**] *A*
$\vdash$ – –
{**1**} *B*

Rule **?** (**case** *C1* *C2*)

Subgoal.**1**

{-**1**} *C2*
{-**2**} *C1*
[-**3**] *A*
$\vdash$ – –
[**1**] *B*

Subgoal.**2**

{-**1**} *C1*
[-**2**] *A*
$\vdash$ – –
{**1**} *C2*
[**2**] *B*

Subgoal.**3**

[-**1**] *A*
$\vdash$ – –
{**1**} *C1*
[**2**] *B*

quantifier rules: skolem, skolem! , and typepred

replace universally quantified variables with constants

{-**1**} *A1*
⊢ – –
{**1**} **Forall** (*s*:*Start*): *B1*(*s*)

Rule **?** (**skolem** ("**s1**"))

quantifier rules: skolem, skolem! , and typepred

replace universally quantified variables with constants

{-**1**} *A1*

⊢ – –

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Rule **?** (**skolem** (**"s1"**))

[-**1**] *A1*

⊢ – –

{**1**} *B1*(*s1*)

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{**1**} *B1*(*s1*)

Rule **?** (**typepred** "*s1*")

{-**1**} *Start*(*s1*)
[-**2**] *A1*
⊢ – –
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{-**1**} *Start*(*s1*)
[-**2**] *A1*
⊢ – –
[**1**] *B1*(*s1*)

{-**1**} **Exists** (*s*:*Start*): *A1*(*s*)
⊢ – –
{**1**} *B1*

Rule **?** (**skolem** "*s0*")

quantifier rules: skolem, skolem! , and typepred

replace universally quantified variables with constants

{-**1**} *A1*
⊢ — —
{**1**} **Forall** (*s*:*Start*)**:** *B1*(*s*)

Rule **?** (**skolem** ("**s1**"))

[-**1**] *A1*
⊢ — —
{**1**} *B1*(*s1*)

Rule **?** (**typepred** "*s1*")

{-**1**} *Start*(*s1*)
[-**2**] *A1*
⊢ — —
[**1**] *B1*(*s1*)

{-**1**} **Exists** (*s*:*Start*)**:** *A1*(*s*)
⊢ — —
{**1**} *B1*

Rule **?** (**skolem** "*s0*")

{-**1**} *A1*(*s0*)
⊢ — —
{**1**} *B1*

quantifier rules and introducing lemmas

{-**1**} *A1*
⊢ – –
{**1**} **Exists** (*n*:**nat**): *B1*(*n*)
Rule **?** (**inst** **1** (*n* **"5"**))

quantifier rules and introducing lemmas

{-**1**} *A1*
⊢ – –
{**1**} **Exists** (*n*:**nat**): *B1*(*n*)
Rule **?** (**inst** **1** (*n* **"5"**))

[-**1**] *A1*
⊢ – –
{**1**} *B1*(**5**)

quantifier rules and introducing lemmas

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Suppose we have:

Fact: **Lemma Exists**(*n*): *P*(*n*)

quantifier rules and introducing lemmas

{-**1**} *A1*
$\vdash$ – –
{**1**} **Exists** (*n*:**nat**): *B1*(*n*)
Rule **?** (**inst 1** (*n* **"5"**))

[-**1**] *A1*
$\vdash$ – –
{**1**} *B1*(**5**)

Suppose we have:

Fact: **Lemma Exists**(*n*): *P*(*n*)

ongoing proof sequent...

{-**1**} **Forall**(*n*): *P*(*n*) $\Rightarrow$ *Q*(*n*)
$\vdash$ – –
{**1**} **Exists**(*n*): *Q*(*n*)

quantifier rules and introducing lemmas

{-**1**} *A1*
⊢ — —
{**1**} **Exists** (*n*:**nat**): *B1*(*n*)
Rule **?** (**inst** **1** (*n* "**5**"))

[-**1**] *A1*
⊢ — —
{**1**} *B1*(**5**)

Suppose we have:

Fact: **Lemma Exists**(*n*): *P*(*n*)

ongoing proof sequent...

{-**1**} **Forall**(*n*): *P*(*n*) ⇒ *Q*(*n*)
⊢ — —
{**1**} **Exists**(*n*): *Q*(*n*)

Rule **?** (**lemma** "*Fact*")

quantifier rules and introducing lemmas

{-**1**} *A1*
⊢ – –
{**1**} **Exists** (*n*:**nat**): *B1*(*n*)
Rule **?** (**inst 1** (*n* **"5"**))

[-**1**] *A1*
⊢ – –
{**1**} *B1*(**5**)

Suppose we have:

Fact: **Lemma Exists**(*n*): *P*(*n*)

ongoing proof sequent...

{-**1**} **Forall**(*n*): *P*(*n*) ⇒ *Q*(*n*)
⊢ – –
{**1**} **Exists**(*n*): *Q*(*n*)

Rule **?** (**lemma** "*Fact*")

{-**1**} **Exists**(*n*): *P*(*n*)
[-**2**] **Forall**(*n*): *P*(*n*) ⇒ *Q*(*n*)
⊢ – –
[**1**] **Exists**(*n*): *Q*(*n*)

quantifier rules and introducing lemmas

{-**1**} *A1*
⊢ – –
{**1**} **Exists** (n:**nat**): *B1*(n)
Rule **?** (**inst** **1** (n "**5**"))

[-**1**] *A1*
⊢ – –
{**1**} *B1*(**5**)

Suppose we have:

Fact: **Lemma Exists**(n): $P(n)$

ongoing proof sequent...

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quantifier rules and introducing lemmas

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1. (**undo** k) undoes proof back to k^{th} level ancestor
2. (**postpone**) mark current goal as pending and move focus to next unproved goal in proof tree
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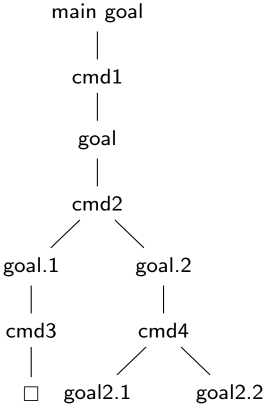

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- (**assert**): simplify
- (**grind**): lift-if, rewrite, and repeatedly simplify

polymorphic theory of automata

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simplemachine[
states, actions: type,
enabled: [actions,states -> bool],
trans: [actions,states -> states],
start: [states -> bool]
]: theory
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reachable_hidden(s,n): recursive bool =
if n = 0 then start(s)
 else (exists a, s1 : reachable_hidden(s1,n -1) and
enabled(a,s1) and s = trans(a,s1))
 endif
measure (lambda s,n: n)

reachable(s): bool = exists n : reachable_hidden(s,n)
```

polymorphic theory of automata

Inv: **var** [*states*-> **bool**]

base(*Inv*) : **bool** = **forall** *s*: *start*(*s*) **implies** *Inv*(*s*)

inductstep(*Inv*) : **bool** = **forall** *s*, *a*: *reachable*(*s*) **and** *Inv*(*s*) **and** *enabled*(*a*,*s*) **implies** *Inv*(*trans*(*a*,*s*))

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inductthm(*Inv*): **bool** = *base*(*Inv*) **and** *inductstep*(*Inv*) **implies** (**forall** *s* : *reachable*(*s*) **implies** *Inv*(*s*))

a distributed algorithm for spreading the min value

```
states: type = [# val: array[I-> nat ] #]

val(i:I, s:states):nat = s`val(i)

s0: states

Start_ax: Axiom Forall(i:I): val(i,s0) >= val(0,s0)

start(s: states): bool = s = s0

actions: datatype begin
 check(i,j:I): check?
end actions
```

a distributed algorithm for spreading the min value

enabled(*a*:*actions*, *s*:*states*):**bool** =
cases *a* **of**
check(*i*,*j*): *true*

trans(*a*, *s*):*states* =
cases *a* **of**
check(*i*,*j*): *s* **with** [*val* := *val*(*s*) **with** [(*i*) := *min*(*val*(*i*,*s*),*val*(*j*,*s*))]]

a distributed algorithm for spreading the min value

count(s): number of agents with value greater than min at state s
following properties capture correctness

1. agent 0 always has the minimum value
2. in every step the count does not increase
3. if count is not 0 then there exists a step for which count decreases

proving correctness of min-spreading algorithm

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MinConst_Inv(*s*)**:bool** = **Forall**(*i*:*I*): *val*(**0**,*s*) $\Leftarrow$ *val*(*i*,*s*)
MinConst: **Lemma Forall** (*s*:*states*): *reachable*(*s*) **Implies** *MinConst_Inv*(*s*)

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$MinConst\_Inv(s)$**:bool** $=$ **Forall**$(i$**:**$I)$**:** $val(\mathbf{0},s) \Leftarrow val(i,s)$

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PVS proof ...

the proof

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("" (lemma "machine_induct")
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 (expand "inductthm")
 (skolem!)
 (split)
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   (expand "MinConst_Inv")
   (expand "start")
   (lemma "Start_ax")
   (skolem!)
   (inst -1 "i!1")
   (assert))
  ("2" (expand "inductstep") (skolem * ("s1" "a"))
   (case "check?(a)")
   (("1" (expand "MinConst_Inv")
     (skolem * ("j1"))
     (copy -3)
     (expand "val" 1)
     (case "i(a) = j1")
     (("1" (inst -2 "i(a)") (inst -5 "j(a)") (grind)) ("2" (inst -1 "j1
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$enabled(a,s)$ **Implies** $count(s) >= count(trans(a,s))$

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Exists $(a{:}actions)$: $count(s) > count(trans(a,s))$

proving correctness of min-spreading algorithm

```
count_rec(i:I, s:states) :recursive nat =
if i = 0 then 0
elsif val(i,s) > val(0,s) then 1 + count_rec(i-1, s)
else count_rec(i-1, s)
endif
measure (lambda(i:I, s:states): i)

count(s:states): nat = count_rec(N,s)
```

proving correctness of min spreading algorithm

count_rec(*i*,*s*): number of agents with value greater than min at state *s* among the first i agents

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stronger version of Non_Increasing lemma

Non_Increasing1: **Lemma Forall** (*s*:*states*,*a*:*actions*): *enabled*(*a*,*s*) **Implies Forall** (*i*:*I*): *count_rec*(*i*,*s*) $>=$ *count_rec*(*i*,*trans*(*a*,*s*))

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IF $j < i(a)$ **THEN** *count_rec*(j,s) $=$ *count_rec*(j, *trans*(a,s))
ELSE *count_rec*(j,s) $=$ **1** $+$ *count_rec*(j, *trans*(a,s)) **ENDIF**

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- heavy weight decision procedures perform acceptably for low-level simplifications but cannot (in general) replace important proof steps
- research direction: for specific application domains such as distributed systems, construct strategies that generate sequences of proof commands from the specification

references

1. PVS system guide http://pvs.csl.sri.com/doc/pvs-system-guide.pdf
   Read chapter 2 for basic instructions about the user interface
2. PVS language http://pvs.csl.sri.com/doc/pvs-language-reference.pdf
3. PVS prover guide http://pvs.csl.sri.com/doc/pvs-prover-guide.pdf