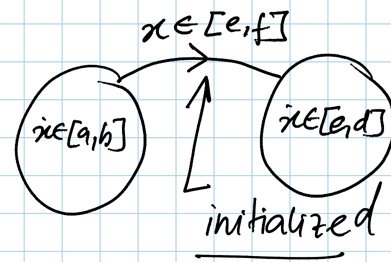
Lecture 12

- Undecidability of CSR for RHA
- RIHA

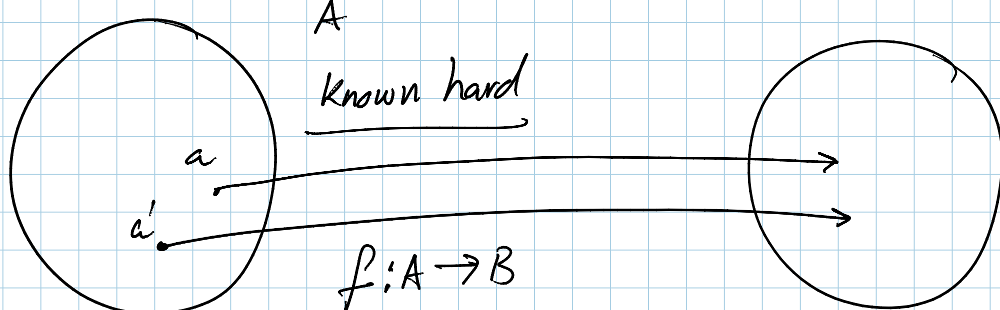


- T. Henzinger 1995
  "What's Decidable about HA?"

Reductions

Problem Class A — Known hard

Problem B — target



Claim: B is also undecidable

A: halting problem for 2-Counter machine ⟶ B: CSR for RHA

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halting problem

Thm: There is no total computable function that takes an arbitrary program and arbitrary input and decides that the program terminates on that input.

$$h(i,x) = 1 \quad \text{if program } i \text{ terminates/halts on input } x$$
$$= 0 \quad \text{otherwise}$$

prog string — i^{th} program

h: is not computable

No computable total function equals h.

Pick an arbitrary ~~al~~ computable function f

Define a partial function $g(i) = 0$ if $f(i,i) = 0$

$= \text{undefined}$ otherwise

f is computable so g is also computable

e: Computeg(i)

{ if $f(i,i) == 0$ then return 0
else loop forever

Diagonalization

$h(e,e) \neq f(e,e) \Rightarrow h \neq f$

$g(e)$ can be 2 things (XOR)

1) $g(e) = 0$ $f(e,e) = 0$ then $h(e,e) = 1$

2) $g(e)$ is undefined $f(e,e) \neq 0$ and $h(e,e) = 0$

TMs can exactly compute the set of computable functions

HP for TMs is undecidable

2-Counter Machines

- finite Program ——— 3 types of instructions
- 2 nonnegative Counters $C, D : \mathbb{N}$ $:= 0$
- halting state $\langle PC, C, D \rangle$

INC IND
DEC DED
JNZC JNZD

Example 1: multiply 3×2

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start: INC
       INC     // C=2
→ loop: IND
       IND
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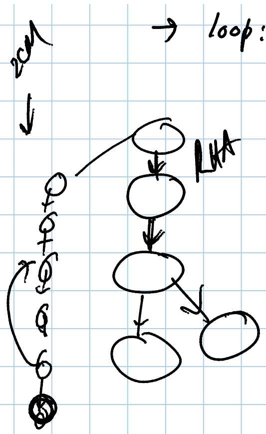
2CM

Example 2: 5 counters with 2CMs

Unique prime factorization of naturals

$n = 2^{k_1} 3^{k_2} 5^{k_3} 7^{k_4} 11^{k_5}$



→ loop: IND
IND
IND // $D=3$; $D=6$
DEC // $C=1$; $C=0$
JNZC loop

$$n = 2^{k_1} 3^{k_2} 5^{k_3} 7^{k_4} 11^{k_5}$$

$k_1, \dots k_5$

$INk_1 \longleftrightarrow n \leftarrow n \times 2$

$DEk_4 \longleftrightarrow n \leftarrow n/7$

$k_1 = 0 \longleftrightarrow n \text{ div } 2$

HP for 2CM is Undecidable.

$c = (1/2)^C$

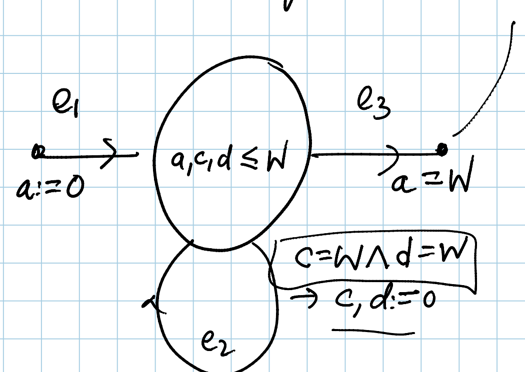
Next INC, DEC, JNZC } how instructions can be emulated by RHA

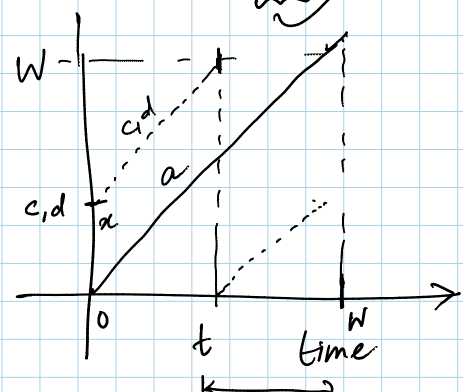
$C = u$ $\quad c = (1/2)^u$

$D = u$ $\quad d = (1/2)^u$

RHA Wrapping automation

continuous variables c, d ~ C, D Caunters of 2CM





Case 1

$c = x \quad d = y \quad x = y$

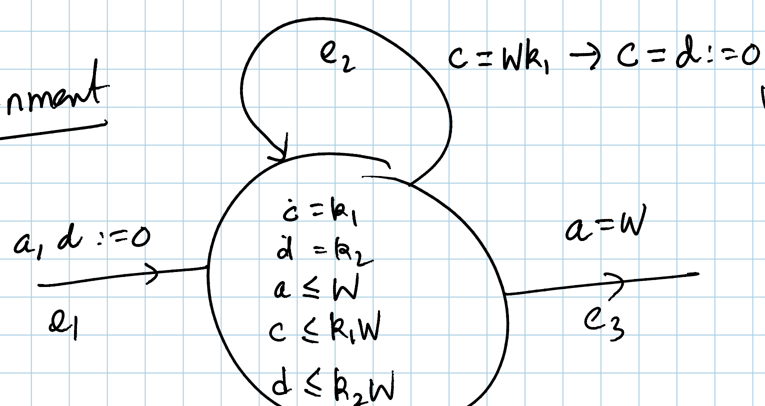
$t = W - x$

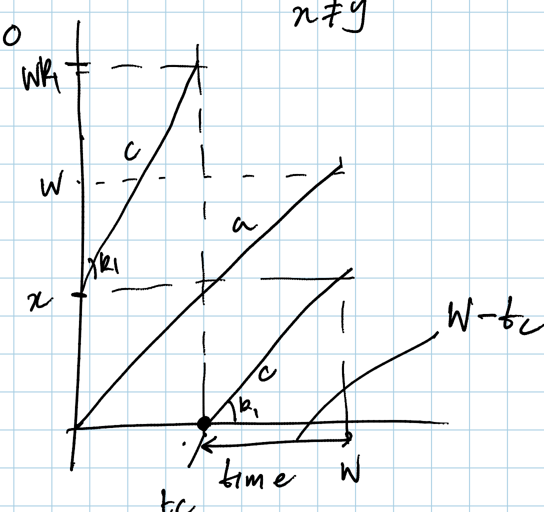
$W - t = W - (W - x)$
$= x$

Case 2

$x \neq y$

Assignment





c entry value x

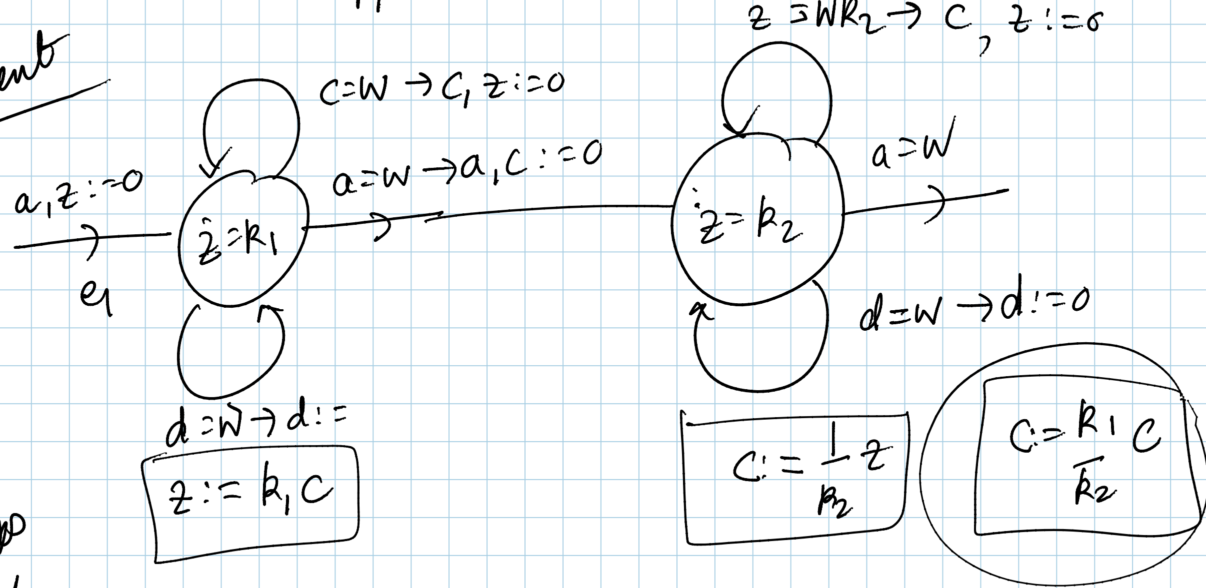
$$t_c = \frac{Wk_1 - x}{k_1}$$

$$W - t_c = \frac{Wk_1 - Wk_1 + x}{k_1}$$

$$k_1(W - t_c) = x$$

$$d = (W - t_c)k_2 = \frac{x}{k_1}k_2$$ if $k_1 = k_2$ $d = x$ = init val of c

decrement



clocks ↓

$$\left.\begin{aligned} c &= |k_1|\left(\frac{k_2}{k_1}\right)^{C} \\ d &= |k_1|\left(\frac{k_2}{k_1}\right)^{D} \end{aligned}\right\}$$ holds only when $a = 0$ or $a = W$

$$INC \sim c \leftarrow \frac{k_1}{k_2}c$$