

DryVR: Data-driven verification and compositional reasoning for automotive systems

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Hybrid modelling: theory vs. reality

Control systems in textbook

Control system in reality

$$\frac{dx}{dt} = f(x, u); u = g(x)$$



"All models are wrong, some are useful"



Gain serenity to accept models as they are



A new view of knowledge in hybrid models

Complete information of switching structure

Executable access to mode dynamics

DryVR's Executable hybrid model





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DryVR model of lane merge



DryVR model semantics





Transition graph Trace: $l_1, t_1, l_2, t_2, ..., l_k$ Black-box simulator Trajectory: $\tau(t)$ Labeled trajectory set: $\langle \tau, l \rangle \in \mathcal{TL}$ Hybrid system $\mathcal{H} = \langle \mathcal{L}, \Theta, G, \mathcal{TL} \rangle$ State: a point in $\mathbb{R}^n \times \mathcal{L}$ $Reach = \{\langle x, l \rangle | \text{ for some } v, t, \langle x, l \rangle \text{ is}$ reachable from $\Theta \}$ Reach | v: all states reachable in vertex v



Proof rules

Bounded model checking

Case studies

Composition for unbounded time analysis

If $Reach|B \subseteq Reach|A$ then



Composition for unbounded time analysis

If $Reach|B \subseteq Reach|A$ then



Reasoning about behavior containment

Trace containment $G_1 \leq G_2$

Trajectory containment $\mathcal{TL}_1 \preccurlyeq \mathcal{TL}_2$

If $\Theta_1 \subseteq \Theta_2$, $G_1 \preccurlyeq G_2$, $\mathcal{TL}_1 \preccurlyeq \mathcal{TL}_2$, then



Simulation-driven bounded verification

Safety problem: given initial set Θ and unsafe set U, decide

Reach $\cap U = \emptyset$?



Simulation-driven bounded verification

Simulation-driven verification for a single vertex v

• Simulate \rightarrow Generalization \rightarrow Check and refine

Discrepancy β bounds distance between neighboring trajectories

 $\|\tau_1(t) - \tau_2(t)\| \le \beta(\tau_1(0), \tau_2(0), t),$

- From a single simulation of $\tau_1(t)$ and discrepancy β overapproximate the reach set from a neighborhood of $\tau_1(0)$
- Earlier approaches use f(x), $\frac{\partial f(x)}{\partial x}$ [Duggirala et al. TACAS 15] [Fan et al. CAV 15-16] inapplicable



Learning discrepancy from data

Global exponential discrepancy function

$$\beta(x_1, x_2, t) = |x_1 - x_2| K e^{\gamma t}$$

For any pair of trajectories au_1 and au_2 in mode



 $\forall t \in [0, T], |\tau_1(t) - \tau_2(t)| \le |\tau_1(0) - \tau_2(0)| K e^{\gamma t}$

Taking logarithm and rearrange:

$$\forall t, \ln \frac{|\tau_1(t) - \tau_2(t)|}{|\tau_1(0) - \tau_2(0)|} \le \gamma t + \ln K$$

Learning linear separators

For a subset $S \subseteq \mathbb{R} \times \mathbb{R}$, a linear separator is a pair $(a, b) \in \mathbb{R}^2$ such that

 $\forall (x, y) \in S, x \le ay + b$

Algorithm:

- 1. Draw k pairs $(x_1, y_1), \dots, (x_k, y_k)$ from S according to D.
- 2. Find $(a, b) \in \mathbb{R}^2$ such that $x_i \leq ay_i + b$ for all $i \in \{1, ..., k\}$.

Proposition [Valiant 84]: Let $\epsilon, \delta \in \mathbb{R}^+$. If $k \ge \frac{1}{\epsilon} \ln \frac{1}{\delta}$ then with probability $1 - \delta$, the above algorithm finds (a, b) such that $err_{\mathcal{D}}(a, b) < \epsilon$.

•
$$err_{\mathcal{D}}(a, b) = \mathcal{D}(\{(x, y) \in S \mid x > ay + b\})$$

Learning discrepancy from data

Solve the LP problem:

min $2c \ln K + c(c+1)\gamma T$ --- Volume of the reach set s.t. $\forall i, j, s, \ln \frac{|\tau_i(t_s) - \tau_j(t_s)|}{|\tau_i(0) - \tau_j(0)|} \le \gamma t_s + \ln K$

1 million testing show 96% accuracy for 10 training trajectories, and >99.9% for 20

Other discrepancy shapes in paper: piece-wise exponential, global polynomial, piece-wise polynomial

Bounded safety algorithm

- 1. Compute reach set from Θ : proceeds on G in a topologically sorted order
- 2. Refinement:
 - Split Θ to smaller sets
 - Split transition time interval to smaller intervals

Guarantee: Assuming that the learned discrepancy function is correct:

- Soundness
- Relative completeness
- Discrepancy has $err_{\mathcal{D}}(a, b) < \epsilon$ with $\geq \frac{1}{\epsilon} \ln \frac{1}{\delta}$ samples



Automotive maneuvers





https://github.com/qibolun/DryVR





Case studies: Engine control

| Model | Time horizo n | Unsafe set | # Refinement | Safe | Run time |
|---------------------------|---------------------|--------------------------|-----------------|------|-------------|
| Auto-passing | 50 | Collision | 4 | ~ | 208s |
| | 50 | Collision | 5 | × | 152s |
| Lane-merge | 50 | Collision | 0 | ~ | 55s |
| | 50 | Collision | 0 | × | 38s |
| Lane-merge- highway | 50 | Collision | 4 | ~ | 197s |
| | 50 | Collision | 0 | × | 21s |
| Powertrain | 80 | Air/Fuel out of bound | 2 | ~ | 217s |
| Automatic transmission | 50 | Engine speed too high | 2 | ~ | 109s |



https://github.com/qibolun/DryVR





Case studies: transmission control

| Model | Time horizo n | Unsafe set | # Refinement | Safe | Run time | |
|---------------------------|---------------------|--------------------------|-----------------|------|-------------|---|
| Auto-passing | 50 | Collision | 4 | ~ | 208s | G |
| | 50 | Collision | 5 | × | 152s | |
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Conclusion

A fresh perspective (DryVR's model) on modeling hybrid systems

- white box transition graph + black box simulator
- Case studies ADAS / AV

Enables types of static-dynamic analysis

- Black-box => discrepancy functions with probabilistic guarantees
- Bounded verification [Sound and relatively complete]
- Proof rules for sequential composition for unbounded time verification and behavior containment

Future: More expressive white boxes, synthesis, monitoring,







Links and references

Textbook picture links:

https://images.google.com/

References :

[Fan et al ATVA 15] Fan, Chuchu, and Sayan Mitra. "Bounded verification with on-the-fly discrepancy computation." International Symposium on Automated Technology for Verification and Analysis. Springer International Publishing, 2015.

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[Valiant 84] Valiant, Leslie G. "A theory of the learnable." Communications of the ACM 27.11 (1984): 1134-1142.

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for your precious time and attention

DryVR model semantics

Transition graph:

• Trace: $l_1, t_1, l_2, t_2, ..., l_k$

Black-box simulator

- Trajectory: $\tau(t)$
- Labeled trajectory set: $\langle \tau, l \rangle \in \mathcal{TL}$

Hybrid system $\mathcal{H} = \langle \mathcal{L}, \Theta, G, \mathcal{TL} \rangle$

- State: a point in $\mathbb{R}^n \times \mathcal{L}$
- Initial states: $\Theta{\times}\,\mathcal{L}_{init}$
- $Reach = \{ \langle x, l \rangle | \text{ for some } v, t, \langle x, l \rangle \text{ is reachable from } \Theta \}$
- Reach|v: all states reachable in vertex v



Learning linear separators (cont.)

For a subset $\Gamma \subseteq \mathbb{R} \times \mathbb{R}$, a linear separator is a pair $(a, b) \in \mathbb{R}^2$ such that

$$\forall (x, y) \in \Gamma, x \le ay + b$$

$$\forall \left(\ln \frac{|\tau_1(t) - \tau_2(t)|}{|\tau_1(0) - \tau_2(0)|}, t \right) \in \Gamma, \ln \frac{|\tau_1(t) - \tau_2(t)|}{|\tau_1(0) - \tau_2(0)|} \le \gamma t + \ln K$$

Proposition [Valiant 84]: Let $\epsilon, \delta \in \mathbb{R}^+$. If $k \ge \frac{1}{\epsilon} \ln \frac{1}{\delta}$ then with probability $1 - \delta$, the above algorithm finds (a, b) such that $err_{\mathcal{D}}(a, b) < \epsilon$.

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DryVR's model of lane merge





DryVR's model of lane merge



