Deductive verification of distributed systems with PVS theorem prover—Part 1

CS141a: Distributed Systems Laboratory

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  - prove that the automaton satisfies the requirements
  - generate verified executable code from automaton through successive refinements

overview of tutorial

- quick introduction to PVS—a theorem prover for high-order logic
  - PVS specification language
  - prover commands
- specifying distributed algorithms in PVS
- proving properties of algorithms using PVS

propositional logic

$$P := \mathit{true} \mid \mathit{false} \mid \neg P_1 \mid P_1 \wedge P_2 \mid P_1 \vee P_2 \mid P_1 \implies P_2 \mid P_1 \iff P_2$$

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many interesting problems can be expressed in propositional logic, e.g., circuit design, hardware verification

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  - harder to decide $\Rightarrow$ fully automatic verification not possible

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- written in LISP, version 4.1 is open source

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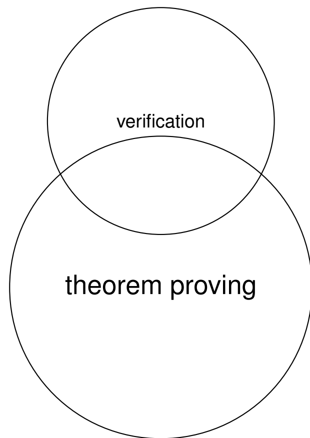

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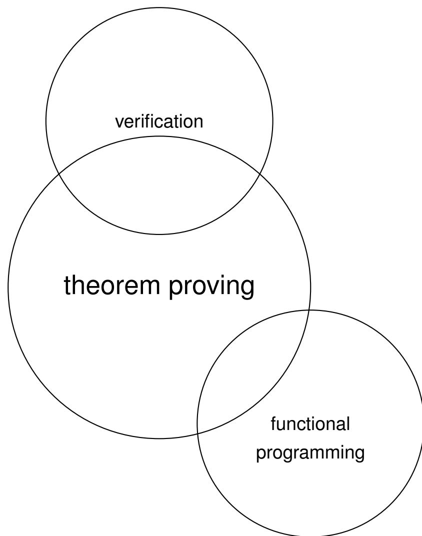

theorem proving and other areas of CS

verification

theorem proving

logic

functional programming

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verification

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PVS

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theorem proving and other areas of CS

verification

proof
methods

theorem proving

PVS

decision
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LISP

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example 1: a theory of stack of integers

Stack: **theory begin**

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top(*c*:*NonEmptyStack*):**nat** = *q*‘*seq*(*length*(*c*)-**1**)

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Stack: type = [# length: nat, seq: [below[length] -> nat ] # ]

NonEmptyStack?(c:Stack): bool = c`length /=  0

NonEmptyStack: type = (NonEmptyStack?)

length(c:Stack):nat = c`length

top(c:NonEmptyStack):nat = q`seq(length(c)-1)

push(c:stack, a:nat):NonEmptyStack =
(# length :=  c`length + 1,
seq :=  seq(c) with [(c`length) :=  a] #)

pop(c:NonEmptyStack):[Stack,nat ]

end Stack
```

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- all assignments and definitions must be type-correct
- typechecking is in general undecidable; PVS generates proof obligations or type correctness conditions (TCCs). E.g., application of $pop(c)$ generates the TCC $NonEmptyStack\mathbf{?}(c)$

some properties of stacks

```
Stack: theory begin

...
c: var Stack
a: var nat

nonempty: lemma forall (c,a): NonEmptyStack?(push(c,a))

idem : lemma forall (c, a): pop(push(c , a))`1 = c

pushpop: lemma forall (c, a): pop(push(c,a))`2 = a

end Stack
```

a polymorphic stack

```
Stack[T:type+]: theory begin

Stack: type = [# length: nat, seq: [below[length] -> T] #]
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end Stack
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inductive definitions and recursive functions

$even(n\mathbf{:}\mathbf{nat})\mathbf{:}$ **inductive** **bool** $= n = \mathbf{0}$ **or** $n > \mathbf{1}$ **and** $even(n\text{-}\mathbf{2})$

inductive definitions and recursive functions

even(*n*:**nat**): **inductive bool** $=$ *n* $=$ **0** **or** *n* $>$ **1** **and** *even*(*n*-**2**)

fact(*n*:**nat**): **recursive nat** $=$ **if** *n* $=$ **0** **then** **1** **else** *n* $*$ *fact*(*n*-**1**) **endif**
measure lambda (*n*:**nat**):*n*

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- mutual recursion not allowed
- domain of the measure function is the same domain as the recursive function being defined and its range must be a well-founded set with a order relation

polymorphic theory of automata

simplemachine[
states, *actions*: **type**,
enabled: [*actions*,*states* -> **bool**],
trans: [*actions*,*states* -> *states*],
start: [*states* -> **bool**]
]: **theory**

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states, actions: type,
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reachable_hidden(s,n): recursive bool =
if n = 0 then start(s)
 else (exists a, s1 : reachable_hidden(s1,n -1) and
enabled(a,s1) and s = trans(a,s1))
 endif
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reachable(s): bool = exists n : reachable_hidden(s,n)
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base(*Inv*) **:** **bool** $=$ **forall** *s***:** *start*(*s*)
implies *Inv*(*s*)

inductstep(*Inv*) **:** **bool** $=$ **forall** *s***,** *a***:** *reachable*(*s*) **and** *Inv*(*s*) **and**
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enabled(*a***,***s*) **implies** *Inv*(*trans*(*a***,***s*))

inductthm(*Inv*)**:** **bool** $=$ *base*(*Inv*) **and** *inductstep*(*Inv*)
implies (**forall** *s* **:** *reachable*(*s*) **implies** *Inv*(*s*))

example: specifying an automaton

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no! push internal nondeterministic choices to (external) choice over actions

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  Values: **type** = [*I* -> **nat**]

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ID:**type** = {**1,2,3,4**}
location:**type** = [*x*:**real**, *y*:**real**]

states: [**#** *pos*:[*ID* -> *location*], *clock*:[*ID* -> **posreal**], *failed*:[*ID* -> **bool**] **#**]

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time_elapse(t:posreal):time_elapse?
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end actions
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  - *fail*?(*a_f3*) returns true

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receive(*i*:*ID*,*m*:*location*):*receive*?
end *actions*

- defines a new type called actions
- a_f3: $actions = fail(\mathbf{3})$ is a constant of type action
  - *fail*?(*a_f3*) returns true
  - *time_elapse*?(*a_f3*) returns false

abstract datatypes

an abstract datatype defines a collection of objects through constructors and recognizers.

actions: **datatype**
fail(*i*:*ID*):*fail*?
time_elapse(*t*:**posreal**):*time_elapse*?
send(*i*:*ID*,*m*:*location*):*send*?
receive(*i*:*ID*,*m*:*location*):*receive*?
end *actions*

- defines a new type called actions
- *a_f3*: *actions* = *fail*(**3**) is a constant of type action
  - *fail*?(*a_f3*) returns true
  - *time_elapse*?(*a_f3*) returns false
  - *i*(*a_f3*) returns 3

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- defines a new type called actions
- *a_f3*: *actions* = *fail*(**3**) is a constant of type action
  - *fail***?**(*a_f3*) returns true
  - *time_elapse***?**(*a_f3*) returns false
  - *i*(*a_f3*) returns 3
  - what is *i*(*time_elapse*(**10**)) ?

defining enabling conditions and transitions

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enabled(a:actions, s:states):bool =
cases a of
fail(i):
not failed(s)(i)
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endcases
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cases *a* **of**
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not $failed(s)(i)$

$send(i,m)$**:**
$pos(s)(i) = m$
...
endcases

$trans(a\!:\!actions,\ s\!:\!states)\!:\!states =$
cases *a* **of**
$time_elapse(t)$**:**
s **with** $[clock := clock(s) + t]$

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send(i,m):
pos(s)(i) = m
...
endcases

trans(a:actions, s:states):states =
 cases a of
time_elapse(t):
s with [clock := clock(s) + t]

fail(i):
s with [failed := failed(s) with [(i) := true]
...
 endcases
```

references

1. PVS system guide `http://pvs.csl.sri.com/doc/pvs-system-guide.pdf`
   Read chapter 2 for basic instructions about the user interface
2. PVS language `http://pvs.csl.sri.com/doc/pvs-language-reference.pdf`
3. PVS prover guide `http://pvs.csl.sri.com/doc/pvs-prover-guide.pdf`