The Wireless Channel

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(Chapter 2 of Course Notes)

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Terminology / Notation

- Bandwidth
- Power content
- Power spectral density
- Energy content
- Decibel notation
Fourier Transform

- If $X(f)$ is fourier transform of $x(t)$ then

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi ft} \, df$$

$X(f)$: Frequency-domain representation
Bandwidth

\[ |X(f)| \]

\[-(f_l + W) \quad -f_l \quad 0 \quad f_l \quad f_l + W\]

\[ W \quad W \quad f \]
Power Content of signal $x(t)$

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} |x(t)|^2 \, dt$$
Power Spectral Density

\[ P_x = \int_{-\infty}^{+\infty} S_x(f) \, df \]

Power content
Energy Content of $x(t)$

$$E_x = \int_{t_1}^{t_2} |x(t)|^2 \, dt$$
Decibel Notation

Power in decibel notation

- Power in dBW = 10 log P ,
  where P is in Watts, log is base-10

- Power in dBm = 10 log P ,
  where P is in milliwatts, log is base-10
A Wireless Link
Digital Communication Link

Transmitter

Channel

Receiver
Noise and Interference

The signal received by the receiver is a composite of

- Signal of interest
- Noise
- Interference

Interference contains information of interest to some other receiver
Additive White Gaussian Noise (AWGN) Process

- **Additive**: Noise added to the signal of interest
- **White**: Noise power spectral density is “flat”, independent of $f$
- **Process**: Noise at each time $t$ is a random variable
- **Gaussian**: $n(t) = \text{noise at time } t$ is a Gaussian random variable
Additive White Gaussian Noise (AWGN)

- Mean 0, variance $\frac{N_0}{2}$
  $$p(N = n) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{n^2}{N_0}}$$

- Power spectral density = $\frac{N_0}{2}$

- Noise power $N$ over bandwidth $W =$
  $$\frac{N_0}{2} (2W) = N_0 W$$

- $W = 10$ MHz, $N_0/2$ is $4 \times 10^{-21}$ : $N = ?$
Signal Propagation

Multipath

mobility
Wireless Channel

- Transmitted signal $x(t)$
- Received signal $y(t)$

\[
y(t) = \sum a_i x(t - \tau_i) + n(t) + i(t)
\]
Wireless Channel

- **Channel gain** \( h(t) \)

\[
y(t) = h(t) \ast x(t) + n(t) + i(t)
\]

\[
h(t) = \sum_{i} a_i \delta(t - \tau_i)
\]
Path Gain

- Path gain = ratio of received power and transmit power

\[ g = \frac{P_r}{P_s} \]

- Path loss = 1 / path gain

- Received power < transmit power
Path Loss Models

- Large scale path loss:
  Captures average channel conditions

- Small scale path loss:
  Captures small scale variations due to fading
Large Scale Path Loss Models

- Free-Space Propagation Model

\[ P_r \propto \frac{P_s}{d^2} \]

\[ P_r = K \frac{P_s}{d^2} \]
Large Scale Path Loss Models

- **Free-Space Propagation Model**

\[ d \geq d_0 \quad : \quad P_r(d) = P_r(d_0) \frac{d_0^2}{d^2} \]

Path loss in decibel (dB)

\[ PL(d) = PL(d_0) + 20 \log \frac{d}{d_0} \]
Large Scale Path Loss Models

- Two-Ray Ground Propagation Model

\[ P_r(d) = P_r(d_0) \frac{d_0^4}{d^4} \]
Large Scale Path Loss Models

- Log-Distance Path Loss Model

Path loss exponent $\alpha$

$$P_r(d) = P_r(d_0) \frac{d_0^\alpha}{d^\alpha}$$
Large Scale Path Loss Models

- Log-Normal Shadowing Model

\[ PL(d) \ [dB] = \overline{PL}(d) \ [dB] + X_\sigma \]

\[ X_\sigma = \text{Gaussian (Normal) random variable with mean 0 and standard deviation } \sigma \]
Small Scale Fading

- Causes of fading:
  - Movement of objects
  - Multipath

- Rayleigh fading model
  Many paths from transmitter to receiver, none being dominant

- Ricean fading model
  A dominant path present, along with other paths
Model versus Reality

- Models don’t capture reality exactly
- Still useful for performance estimation
Principle of Reciprocity

- Same channel gain in both directions at a given point of time

- Asymmetry in practice because
  - Transmissions in two directions occur at different times
    - Can still assume symmetry if channel varies slowly
  - Different hardware implementations at two ends
Transmission must use the appropriate band of the spectrum

Modulation can facilitate the use of appropriate band
Modulation & Demodulation

Example:

Binary pulse amplitude modulation (Binary PAM)
Binary PAM : Modulation

- Pulse $b(t)$ of duration $T$
- Consider the very first bit
- Baseband signal:
  
  bit 0: $s_0(t) = -b(t)$
  
  bit 1: $s_1(t) = b(t)$
Binary PAM: Modulation

- Baseband signal “superimposed” on the carrier
- Multiply the baseband signal by carrier

Modulated signal

\[ w_i(t) = s_i(t) \cos(2\pi f_c t) \]

\[ = A_i b(t) \cos(2\pi f_c t) \]

\[ A_i = \begin{cases} -1 & \text{for } i = 0 \\ 1 & \text{for } i = 1 \end{cases} \]
Received Signal

- Assume single path from transmitter to receiver with zero delay

- AWGN channel with noise power spectral density $N_0/2$

$$r(t) = h w_i(t) + n(t)$$
Energy-per-Bit
in received signal of interest

\[ E_b = \int_0^T (h w_i(t))^2 \, dt \]

\[ = h^2 \int_0^T b^2(t) \cos^2(2\pi f_c t) \, dt \]
Demodulation

\[ z = \frac{1}{\sqrt{E_b}} \int_0^T r(t) h b(t) \cos(2\pi f_c t) \, dt \]

\[ = \frac{1}{\sqrt{E_b}} \int_0^T h w_i(t) h b(t) \cos(2\pi f_c t) \, dt + \]

\[ \frac{1}{\sqrt{E_b}} \int_0^T n(t) h b(t) \cos(2\pi f_c t) \, dt \]
Demodulation

Can be shown that

\[ z = A_i \sqrt{E_b} + m \]

where \( m \) is Gaussian random variable with 0 mean and variance \( N_0/2 \)
Demodulation

Optimal decision rule assuming that 0 and 1 are transmitted with equal probability:

Decide that the transmitter sent

- 0 if $z < 0$
- 1 if $z > 0$
- 0 or 1 if $z = 0$

\[ \begin{array}{ccc}
-\sqrt{E_b} & 0 & \sqrt{E_b} \\
\end{array} \]
Error Probability

\[ P_b = \int_{-\sqrt{E_b}}^{\sqrt{E_b}} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{u^2}{2(N_0/2)}} \, du = Q\left(\frac{\sqrt{E_b}}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \]

\[ Q(u) = \int_{u}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} \, du \]
Binary PAM

- Errors occur during transmission
- Packet error probability function of bit error probability and packet size

- Error probability can be reduced by increasing energy per bit
  - higher transmit power and/or
  - lower transmission rate
    - Trade-off between transmission rate & error probability

Similar observations hold for other modulation schemes
Error Control Codes

- As we saw, errors may occur during transmission over the wireless channel.

- Error control codes (ECC) can be used to detect and/or correct such errors.

- Error control capabilities of a code depend on the redundancy introduced by the code.

- \((n,k)\) code: \(k\) bits of data, \(n-k\) checkbits.
Hamming distance

- Code = set of codewords

- Hamming distance between two codewords
  = number of bits in which they differ

- Distance of a code
  = minimum Hamming distance between two codewords
(7,4) Single Error Correcting (SEC) Code

Rate 4/7

Distance 3

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<th>Codeword</th>
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<tr>
<td>0000</td>
<td>0000 000</td>
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<tr>
<td>0001</td>
<td>0001 011</td>
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<tr>
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<td>0010 111</td>
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<td>1110 100</td>
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<tr>
<td>1111</td>
<td>1111 111</td>
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(8,4) Single Error Correcting-Double Error Detecting (SEC-DED) Code

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<td>1111 1111</td>
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Rate $4/8 = 1/2$

Distance 4
The (7,4) Code

- The code may be used as a SEC code
  - To correct single bit errors
  - More than 1 error may result in a decoding error or an undetected error

- Alternatively, the code may be used to detect up to 2 bit errors (but correct no errors)
  - More than 2 errors may not be detected
The (8,4) Code

- The code may be used as a SEC-DED code
  - To correct single bit errors
  - Detect 2 bit errors
    (and be able to differentiate between 1 and 2 errors)
  - More than 2 errors may result in a decoding error or an undetected error

- Alternatively, the code may be used to detect up to 3 bit errors (but correct no errors)
  - More than 3 errors may not be detected
Error Control Codes

- ECC may not be able to correct or detect all errors
- Higher layers cannot rely on lower layers to detect/correct all errors
- Different protocol layers incorporate different ECC mechanisms, depending on the reliability requirements
- Ultimate responsibility for reliability at the application layer
Error Control Codes

- Error probability a function of transmit power, which is constrained

- ECC incurs overhead, which reduces the effective data rate

- What is the best possible reliable rate?
Capacity

- AWGN channel with bandwidth $W$

\[
C = W \log_2 \left( 1 + \frac{P}{N_0 W} \right)
\]
Capacity

\[ C = W \log_2 (1 + SINR) \]

\[ SINR = \frac{P}{I + N_0 W} \]
Wireless “Link” and Transmission “Range”

- When does a link “exist” or “breaks”?
  - Function of parameters such as transmit rate, power, reliability requirements

- Transmission range?
SINR-Threshold Model

- Errors a non-deterministic phenomenon

- Deterministic approximation:

  If SINR exceeds a threshold, assume reliable transmission