Wireless Network Capacity

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Wireless Networks

- Why use multi-hop routes to deliver data?
- Is this optimal?
- What’s the “best” performance achievable?
- Capacity analysis can help answer such questions
Capacity of Wireless Networks
[Gupta-Kumar]

- Exact capacity is difficult to analyze
- Easier to analyze “trends” in capacity
- We will consider a few such results
- Two network models:
  - Arbitrary network
  - Random network
Network Model
Arbitrary Networks: Interference Constraint: “Protocol Model”

- $d(i, j) = \text{distance between nodes } i \& j$

- Parameter $\Delta$ (constant)

- $i \rightarrow j$ (node $i$ is transmitting to node $j$)

- If node $k$ transmits simultaneously (interference) then $i \rightarrow j$ transmission is reliable iff

$$d(k, j) \geq (1 + \Delta)d(i, j), \quad \Delta > 0$$
Protocol Model

\[ d(k, j) \geq (1 + \Delta)d(i, j), \quad \Delta > 0 \]
Arbitrary Networks: Interference Constraint

- Interference model is an approximation of reality

- Approximation adequate to derive results on “trends” in capacity as a function of number of nodes

  ➔ How does network capacity change when number of nodes is increased?

- How to define capacity?
Arbitrary Networks

- $n =$ number of nodes in a unit area
- $W =$ rate of successful transmission
  (interpreted as a measure of “bandwidth”)

- Arbitrary
  - Place $n$ nodes wherever desired
  - Choose any flows as desired
    (each flow has a source & a destination)
  - Schedule transmissions as desired
    (under interference constraint)
Arbitrary Network

- Placing transmitter-receiver close to each other, n/2 nodes can transmit reliably simultaneously

⇒ Aggregate throughput can increase linearly with n
Arbitrary Network

- When transmitter-receiver placed close, information doesn’t travel very far
- Throughput alone is not interesting metric

- Throughput * Distance a more interesting metric

  - Analogy: Work = Force * Distance
Arbitrary Networks

- Transport capacity = $\max \sum \text{Throughput} \times \text{Distance}$
  - Bit-meter/second

- Transmitter-receiver too close
  - small distance
  - reduced transport capacity

- Transmitter-receiver too far
  - lower spatial reuse due to interference
  - reduced transport capacity
Arbitrary Networks

- How does transport capacity scale with \( n \)?
- Results ...
Order Notation

- f(n) is $O(g(n))$ if there exists a constant $d$ such that, for large enough $n$,
  $$f(n) < d g(n)$$

- Examples:

  5 log $n + 2000$ is $O(n)$

  2$n$ is NOT $O(\log n)$
Order Notation

- $f(n)$ is $\Omega(g(n))$ if $g(n)$ is $O(f(n))$

- Examples:

  - $n$ is $\Omega(5 \log n + 2000)$
  - $\log n$ is NOT $\Omega(n)$
Order Notation

- $f(n)$ is $\Theta(g(n))$ if
  
  $f(n)$ is $O(g(n))$ \&
  $f(n)$ is $\Omega(g(n))$

- Examples:
  
  $\log n$ is NOT $\Theta(n)$
  (5000 $n + 12$) is $\Theta(n)$
Arbitrary Networks

How does transport capacity scale with $n$?

Results

- Upper bound $O(W \sqrt{n})$
- Lower bound construction $\Omega(W \sqrt{n})$

$\Rightarrow$ Transport capacity is $\Theta(W \sqrt{n})$ bit-meters/second
Upper Bound: Arbitrary Networks

- Assume slots of duration $\tau$

- One flow at each of the $n$ nodes

- $\lambda = $ Average rate (average over $n$ flows)

- $\bar{L} = $ average distance between transmitter-receiver of a bit (averaged over all the bits)

- $\lambda n \bar{L} = $ Transport capacity
Upper Bound: Arbitrary Networks

- Consider bit $b$, $1 \leq b \leq \lambda n$

- $h(b) =$ number of hops on route used for bit $b$

- $r^h_b =$ distance between endpoints of $h$-hop of bit $b$

\[
\sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} r^h_b \geq \lambda n \bar{L} \quad (1)
\]
Upper Bound: Arbitrary Networks

- \( H = \) total number of bits transmitted in 1 second by all \( n \) nodes combined
- Each transmission uses two hosts

\[
H \leq \frac{Wn}{2} \quad (2)
\]
\begin{align*}
d(C, B) &\geq (1 + \Delta)d(A, B) \\
d(A, D) &\geq (1 + \Delta)d(C, D)
\end{align*}

The above two inequalities imply that
\begin{align*}
d(C, B) - d(A, B) &\geq \Delta d(A, B) \\
d(A, D) - d(C, D) &\geq \Delta d(C, D)
\end{align*}

Adding the above two expressions together, we get
\begin{align*}
d(C, B) - d(C, D) + d(A, D) - d(A, B) &\geq \Delta(d(A, B) + d(C, D))
\end{align*}

Now, by triangle inequality, we have
\begin{align*}
d(B, D) + d(C, D) &\geq d(C, B) \quad \Rightarrow \quad d(B, D) \geq d(C, B) - d(C, D) \\
d(B, D) + d(A, B) &\geq d(A, D) \quad \Rightarrow \quad d(B, D) \geq d(A, D) - d(A, B)
\end{align*}

From the above three inequalities, we can conclude that
\begin{align*}
2d(B, D) &\geq d(C, B) - d(C, D) + d(A, D) - d(A, B) \geq \Delta(d(A, B) + d(C, D))
\end{align*}
Protocol Model

- For transmissions below to be both reliable:
  
  \[ d(B, D) \geq \frac{\Delta}{2} (d(A, B) + d(C, D)) \]

- Each transmission “consumes” area of radius \((\Delta/2)\)distance around the receiver
Since any part of the unit area can be consumed by at most $W$ bits in 1 second:

\[
\sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} \frac{\pi \Delta^2 (r^h_b)^2}{4} \leq W \tag{3}
\]

Rewriting this inequality:

\[
\sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} \frac{1}{H} (r^h_b)^2 \leq \frac{4W}{\pi \Delta^2 H} \tag{4}
\]

It can be shown that:

\[
\left( \sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} \frac{1}{H} r^h_b \right)^2 \leq \sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} \frac{1}{H} (r^h_b)^2 \tag{4}
\]
Since any part of the unit area can be consumed by at most \( W \) bits in 1 second:

\[
\sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} \frac{\pi \Delta^2 (r_b^h)^2}{4} \leq W \quad (3)
\]

Rewriting this inequality:

\[
\sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} \frac{1}{H} (r_b^h)^2 \leq \frac{4W}{\pi \Delta^2 H} \quad (4)
\]

It can be shown that:

\[
\left( \sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} \frac{1}{H} r_b^h \right)^2 \leq \sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} \frac{1}{H} (r_b^h)^2 \quad (4)
\]

Using (1), (2), and rearranging terms:

\[
\sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} r_b^h \leq \sqrt{\frac{4WH}{\pi \Delta^2}} \quad (5)
\]

From (1), (2), (5):

\[
\lambda n \bar{L} \leq \sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} r_b^h \leq \sqrt{\frac{4WH}{\pi \Delta^2}} \leq W \sqrt{\frac{2n}{\pi \Delta^2}}
\]
Transport Capacity of Arbitrary Networks: Upper Bound

\[ \lambda n \bar{L} \leq W \sqrt{\frac{2n}{\pi \Delta^2}} \]

Although we assumed that each transmission uses all the bandwidth, this result (and a similar proof) applies even if the spectrum is divided into multiple channels, \textit{provided that} each node can use all available channels simultaneously.

\[ \Rightarrow \text{Transport capacity is } O(W \sqrt{n}) \]
Transport Capacity of Arbitrary Networks: Constructive Lower Bound

- Divide the unit area into \( \frac{n}{8} \) square cells

- Side of each cell of length \( l = \sqrt{\frac{8}{n}} \)

- Place 8 nodes in each cell (as shown on next slide)
\[ l = 2r (1 + 2\Delta) \]
$r (1 + \Delta)$
Transport Capacity of Arbitrary Networks: Constructive Lower Bound

- $n/2$ transmissions at rate $W$
- Distance between transmitter-receiver = $r$

\[
 r = \frac{l}{2(1+2\Delta)} = \frac{1}{(1+2\Delta)} \sqrt{\frac{2}{n}}
\]

- $\Sigma$ Throughput-distance =

\[
 \frac{n}{2} W r = \frac{W}{(1+2\Delta)} \sqrt{\frac{n}{2}} \text{ bit-meters/sec}
\]
Transport Capacity of Arbitrary Networks

- \[ \Sigma \text{ Throughput-distance} = \frac{n}{2} W r = \frac{W}{(1+2\Delta)} \sqrt{\frac{n}{2}} \text{ bit-meters/sec} \]

- Transport capacity \[ \Omega (W \sqrt{n}) \text{ : lower bound} \]

Lower bound & upper bound

- \[ \Theta (W \sqrt{n}) \text{ bit-meters/second} \]
Random Network

- Nodes placed randomly over unit area:
  Expected number of nodes in area $A = nA$

- Each node picks as destination the node closest to a randomly chosen location (uniform distribution)

- Transmission “range” = $r$

- $i \rightarrow j$ reliable if interferers not too close to $j$

$$d(k, j) \geq (1 + \Delta)r$$
Random Network: Throughput Capacity

- Suppose all flows achieve identical rate $\lambda$

- Throughput Capacity = Maximum $n \lambda$ bits/second

- Distance not included in *throughput capacity*, since average distance pre-determined by random topology formation

- Throughput capacity of random networks

  $\Theta \left( W \sqrt{\frac{n}{\log n}} \right)$ bits/second
Multiple Channels
Multi-Channel Environments

Available spectrum

Spectrum divided into channels

1 2 3 4 ... c
Multiple Channels

IEEE 802.11 in ISM Band
Shared Access: Time & Spectrum

One Channel

Two Channels
Why Divide Spectrum into Channels?

- **Manageability:**
  - Different networks on different channels avoids interference between networks

- **Contention mitigation:**
  - Fewer nodes on a channel reduces channel contention
Why Divide Spectrum into Channels?

- Lower transmission rate per channel
  - Slower hardware (simpler, cheaper)

- Reducing impact of bandwidth-independent overhead
Consider A transmitting Data to B, followed by Ack from B

\[
T = \frac{D A T A / R}{(D A T A + A C K) / R + 2\tau} \times R = \frac{D A T A \times R}{D A T A + A C K + 2\tau R}
\]

efficiency \(= \frac{T}{R} = \frac{D A T A}{D A T A + A C K + 2\tau R}\)
- $c$ channels

$$T = c \times \frac{DATA/(R/c)}{(DATA + ACK)/(R/c) + 2\tau} \times (R/c) = \frac{DATA \times R}{DATA + ACK + 2\tau(R/c)}$$

- Efficiency = $T/R$
Data = 4000 bits, Ack = 100 bits, \( \tau = 1 \mu s \)
Capacity of Multi-Channel Networks

- Earlier results apply when each node can tune to all available channels *simultaneously*

- Now consider the case when this is not feasible
Interfaces & Channels

- An interface can only use one channel at a time

CAVEAT: In the course notes, the total bandwidth is specified as $W$, with per-channel bandwidth $W/c$. 
Multiple Interfaces

- Decreasing hardware cost allows for multiple interfaces
- \( m \) interfaces per node
Practical Scenario

$m < c$

A host can only be on a subset of channels

$c - m$ unused channels at each node
Interface Constraint

- Throughput is limited by number of interfaces in a neighborhood

\[ M \text{ nodes in the “neighborhood”} \]
\[ \Rightarrow \text{total throughput} \leq M \times m \times W \]

Interfaces as a resource
in addition to spectrum, time and space
Impact on Routing

- $m = 1$, $c = 1$ or $2$ (compare scenario below)
Multi-Channel Throughput Capacity of Random Networks

Channels (c/m) →

Capacity
CAVEAT: In the course notes, the total bandwidth is specified as $W$, with per-channel bandwidth $W/c$.

Therefore, the results in the graph on previous slides are obtained by replacing $W$ in the formula in the course notes by $(Wc)$. 
Multi-Channel Networks

- When number of channels “not large”, possible to achieve capacity improvement with more channels (c), even if hardware (m) is limited

- Similar result for arbitrary networks (see course notes)
Improving Capacity

Wireless network capacity can be improved by adding infrastructure
- Base stations
- Wired network connecting the base stations (wired links do not compete with wireless)

Mobility may also be exploited to improve capacity

→ Reduce wireless channel usage, by exploiting infrastructure and mobility