ECE 462 (Spring 2005) Homework1 Solutions

1. $f(x0, x1, \ldots, x9) = x0' x1x2' \ldots x9 + x0x1'x2 \ldots x9' = \sum(341,682)$

2. (a) $f(x, y, z) = \sum(0, 3,6)$
Canonical sum:
$f(x, y, z) = x'y'z' + x'yz + xyz'$
Canonical product:
$f(x, y, z) = (x + y + z')(x + y' + z)(x' + y + z)(x' + y + z')(x' + y' + z')$

(b) $f(x, y, z) = \prod(1, 2,7)$
Canonical sum:
$f(x, y, z) = x'y'z' + x'yz + xy'z' + xy'z + xyz'$
Canonical product:
$f(x, y, z) = (x + y + z')(x + y' + z)(x' + y' + z')$

3.
(a)
$F' = (a + bc)' = a' (b' + c') = a'b' + a'c'$

Prove correctness: $f + f' = a + bc + a'b' + a'c' = (a + a'b') + bc + a'c' = a + b' + bc + a'c'$
$= a + (b' + bc) + a'c' = a + b' + c + a'c' = a + b' + (c + a'c') = a + b' + c + a' = 1$

(b)
$F' = ((a + b)(a'c + d))' = a'b' + (a + c') d' = a'b' + ad' + c'd'$

Prove correctness: $f + f' = (a + b)(a'c + d) + a'b' + ad' + c'd' = 0 + ad + a'bc + bd + a'b' +$
$ad' + c'd' = a (d + d') + a'bc + bd + a'b' + c'd' = 1 + a'bc + bd + a'b' + c'd' = 1$

4.
(i) $f(x, y, z) = x'y + y'z$
Karnaugh map:

z \ xy	00	01	11	10
0	0	1	0	0
1	1	1	0	1

(ii) Canonical sum:
$f(x, y, z) = x'y'z + x'yz' + x'yz + xy'z$

(iii) Canonical product:

$f(x, y, z) = (x + y + z)(x' + y + z)(x' + y' + z)(x' + y' + z')$

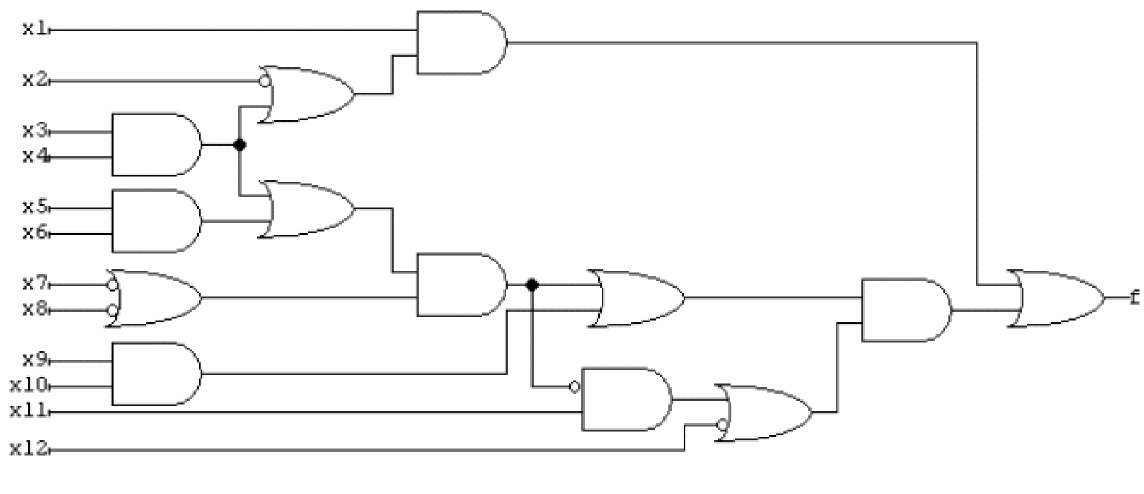
(iv) Reed-Muller canonical form:

$f(x, y, z) = x'y + y'z = x'y \wedge y'z$ (because cube for x'y and y'z are non-overlapping) =

$= (x \wedge 1)y \wedge (y \wedge 1)z = xy \wedge y \wedge yz \wedge z$ (^ is XOR $\oplus$)

5. (Notice there are multiple correct solutions to this problem)

Problem 3.4 (b)



Problem 3.4 (c)

$f(x1, x2, \ldots, x12) = x1(x2' + x3x4) + \{[(x3x4 + x5x6)(x7' + x8') + x9x10]\}\{x11[(x3x4 + x5x6)(x7' + x8')]' + x12'\}$

6.

$f = x1x2 + x1x2x5 + x1x2x4 + x3x2 + x3x4 + x3x5 + x5 + x3x4$

$= x1x2 + x2x3 + x3x4 + x5$

7.

Problem 6.1 (f) Part (i)

wx (v=0)

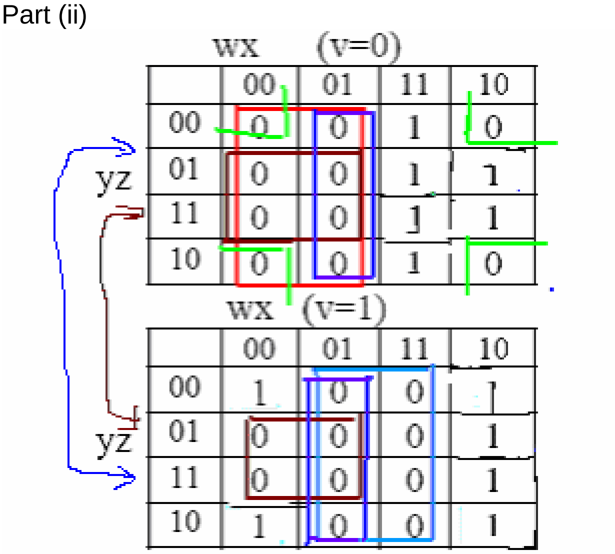
yz	00	01	11	10
00	0	0	1	0
01	0	0	1	1
11	0	0	1	1
10	0	0	1	0

wx (v=1)

yz	00	01	11	10
00	1	0	0	1
01	0	0	0	1
11	0	0	0	1
10	1	0	0	1

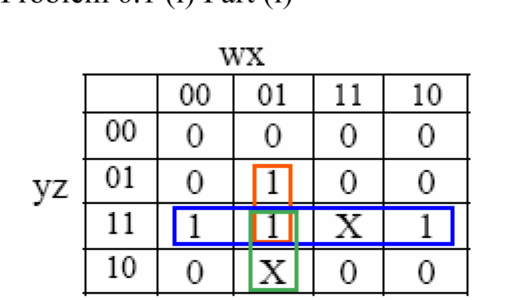
Prime implicants: v’wx, vwx’, v’wz, wx’z, vx’z’.
Complete sum: f = v’wx + vwx’ + v’wz + wx’z + vx’z’
Essential Prime Implicants: vx’z’, v’wx
Minimal sum: f = v’wx + wx’z + vx’z’ (there are more than one correct answers)

Part (ii)



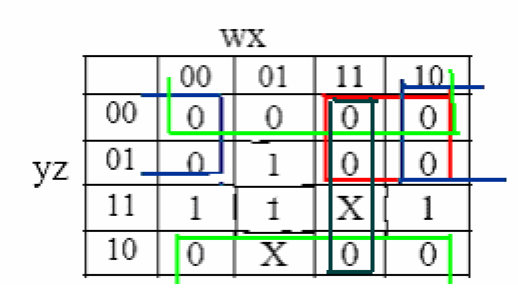
The Prime implicates: x’ + v’, w + v, w + z’, w + x’, and z + x + v
Minimal Product: f = (x’ + v’)(w + v)(w + z’)(z + x + v)

Problem 6.1 (i) Part (i)



Prime implicants: yz, w’xz, w’xy
Complete sum: f = yz + w’xz + w’xy
Essential Prime Implicants: yz, w’xz
Minimal sum: f = yz + w’xz

Part (ii)



Prime implicates: z, w’ + y , w’ + x’, and x + y
Minimal Product: f = z(w’ + y)(x + y)

8.

for the function f(v,w,x,y,z) =
Σ(4,5,8,9,12,13,14,15,16,17,20,21,22,23,24,25,26,28,29,30,31)

	v	w	x	y	z	
4	0	0	1	0	0	X
8	0	1	0	0	0	X
16	1	0	0	0	0	X
5	0	0	1	0	1	X
9	0	1	0	0	1	X
12	0	1	1	0	0	X
17	1	0	0	0	1	X
20	1	0	1	0	0	X
24	1	1	0	0	0	X
13	0	1	1	0	1	X
14	0	1	1	1	0	X
21	1	0	1	0	1	X
22	1	0	1	1	0	X
25	1	1	0	0	1	X
26	1	1	0	1	0	X
28	1	1	1	0	0	X
15	0	1	1	1	1	X
23	1	0	1	1	1	X
29	1	1	1	0	1	X
30	1	1	1	1	0	X
31	1	1	1	1	1	X
(4,5)	0	0	1	0	-	X
(4,12)	0	-	1	0	0	X
(4,20)	-	0	1	0	0	X
(8,9)	0	1	0	0	-	X
(8,12)	0	1	-	0	0	X
(8,24)	-	1	0	0	0	X
(16,17)	1	0	0	0	-	X
(16,20)	1	0	-	0	0	X
(16,24)	1	-	0	0	0	X

	v	w	x	y	z	
(5,13)	0	-	1	0	1	X
(5,21)	-	0	1	0	1	X
(9,13)	0	1	-	0	1	X
(9,25)	-	1	0	0	1	X
(12,13)	0	1	1	0	-	X
(12,14)	0	1	1	-	0	X
(12,28)	-	1	1	0	0	X
(17,21)	1	0	-	0	1	X
(17,25)	1	-	0	0	1	X
(20,21)	1	0	1	0	-	X
(20,22)	1	0	1	-	0	X
(20,28)	1	-	1	0	0	X
(24,25)	1	1	0	0	-	X
(24,26)	1	1	0	-	0	X
(24,28)	1	1	-	0	0	X
(13,15)	0	1	1	-	1	X
(13,29)	-	1	1	0	1	X
(14,15)	0	1	1	1	-	X
(14,30)	-	1	1	1	0	X
(21,23)	1	0	1	-	1	X
(21,29)	1	-	1	0	1	X
(22,23)	1	0	1	1	-	X
(22,30)	1	-	1	1	0	X
(25,29)	1	1	-	0	1	X
(26,30)	1	1	-	1	0	X
(28,29)	1	1	1	0	-	X
(28,30)	1	1	1	-	0	X
(15,31)	-	1	1	1	1	X
(23,31)	1	-	1	1	1	X
(29,31)	1	1	1	-	1	X
(30,31)	1	1	1	1	-	X

	v	w	x	y	z	
(4,5,12,13)	0	-	1	0	-	X
(4,5,20,21)	-	0	1	0	-	X
(4,12,20,28)	-	-	1	0	0	X
(8,9,12,13)	0	1	-	0	-	X
(8,9,24,25)	-	1	0	0	-	X
(8,12,24,28)	-	1	-	0	0	X
(16,17,20,21)	1	0	-	0	-	X
(16,17,24,25)	1	-	0	0	-	X
(16,20,24,28)	1	-	-	0	0	X
(5,13,21,29)	-	-	1	0	1	X
(9,13,25,29)	-	1	-	0	1	X
(12,13,14,15)	0	1	1	-	-	X
(12,13,28,29)	-	1	1	0	-	X
(12,14,28,30)	-	1	1	-	0	X
(17,21,25,29)	1	-	-	0	1	X
(20,21,22,23)	1	0	1	-	-	X
(20,21,28,29)	1	-	1	0	-	X
(20,22,28,30)	1	-	1	-	0	X
(24,25,28,29)	1	1	-	0	-	X
(24,26,28,30)	1	1	-	-	0	
(13,15,29,31)	-	1	1	-	1	X
(14,15,30,31)	-	1	1	1	-	X
(21,23,29,31)	1	-	1	-	1	X
(22,23,30,31)	1	-	1	1	-	X
(28,29,30,31)	1	1	1	-	-	X

	v	w	x	y	z	
(4,5,12,13,20, 21,28,29)	-	-	1	0	-	
(8,9,12,13,24, 25,28,29)	-	1	-	0	-	
(16,17,20,21, 24,25,28,29)	1	-	-	0	-	
(12,13,14,15, 28,29,30,31)	-	1	1	-	-	
(20,21,22,23, 28,29,30,31)	1	-	1	-	-	

F=vwz'+xy'+wy'+vy'+wx+vx