ECE 462 Homework 3 Solutions (Spr 2005)

1. Problem 6.13 (a) From Homework 2:

$f1 = wxy + wx'y'z$

$f2 = x'z' + wxz + xy'z$

$f3 = w'z' + wxyz + w'x'y'$

MoPI's = wxy, $w'y'z$, $x'z'$, wxz, $xy'z$, $wxyz$, $w'x'y'$, $w'xy'z$, and $w'x'y'z$

		f1				f2							f3					
		1	5	14	15	0	2	5	8	10	13	15	0	1	2	8	10	15
14, 15	wxy			X	X													
1, 5	$w'y'z$	X	X															
0,2,8,10	$x'z'$					X	X		X	X			X		X	X	X	
13, 15	wxz										X	X						
5, 13	$xy'z$							X			X							
15	$wxyz$				X							X						X
0, 1	$w'x'y'$												X	X				
5	$w'xy'z$		X					X										
1	$w'x'y'z$	X												X				

$f1 = wxy +$

$f2 = x'z' +$

$f3 = x'z' + wxyz$

Eliminate columns covered by essential MoPI's

		f1		f2			f3	Cg	Cd
		1	5	5	13	15	1		
1,5	$w'y'z$	X	X					1	4
13,15	wxz				X	X		1	4
5,13	$xy'z$			X	X			1	4
15	$wxyz$					X		0	1
0,1	$w'x'y'$						X	1	4
5	$w'xy'z$		X	X				1	5,6
1	$w'x'y'z$	X					X	1	5,6

For f1, (1,5) has lower cost than (5) + (1), So choose (1,5)
For f2, the combination with the lowest cost is (15) + (5,13)
For f3, (1,4) has lower cost than (1)

So $f1 = wxy + w'y'z$

$f2 = x'z' + xy'z + wxyz$

$f3 = x'z' + wxyz + w'x'y'$

2. Problem 6.17

Part (i) Use Quine-McClusky Tabulation Method to determine MoPI's.

	w	x	y	z	f1	f2	f3	
0	0	0	0	0	0	1	1	✓
1	0	0	0	1	1	1	0	✓
2	0	0	1	0	0	0	1	✓
* 8	1	0	0	0	1	1	1	
3	0	0	1	1	0	0	1	✓
5	0	1	0	1	1	1	0	✓
6	0	1	1	0	0	0	1	✓
9	1	0	0	1	0	0	1	✓
10	1	0	1	0	0	0	1	✓
12	1	1	0	0	1	0	0	✓
7	0	1	1	1	1	0	0	✓
11	1	0	1	1	0	0	1	✓
13	1	1	0	1	1	0	0	✓
* 14	1	1	1	0	1	1	1	
15	1	1	1	1	1	0	0	✓

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2 Continued:

	W	X	Y	Z	f_1	f_2	f_3		
*(0,1)	0	0	0	–	0	1	0		
(0,2)	0	0	–	0	0	0	1	✓	
*(0,8)	–	0	0	0	0	1	1		
*(1,5)	0	–	0	1	1	1	0		
(2,3)	0	0	1	–	0	0	1	✓	
(2,6)	0	–	1	0	0	0	1	✓	
(2,10)	–	0	1	0	0	0	1	✓	
(8,9)	1	0	0	–	0	0	1	✓	
(8,10)	1	0	–	0	0	0	1	✓	
*(8,12)	1	–	0	0	1	0	0		
(3,11)	–	0	1	1	0	0	1	✓	
(5,7)	0	1	–	1	1	0	0	✓	⇒
(5,13)	–	1	0	1	1	0	0	✓	
(6,14)	–	1	1	0	0	0	1	✓	
(9,11)	1	0	–	1	0	0	1	✓	
(10,11)	1	0	1	–	0	0	1	✓	
(10,14)	1	–	1	0	0	0	1	✓	
(12,13)	1	1	0	–	1	0	0	✓	
(12,14)	1	1	–	0	1	0	0	✓	
(7,12)	–	1	1	1	1	0	0	✓	
(13,15)	1	1	–	1	1	0	0	✓	
(14,15)	1	1	1	–	1	0	0	✓	

2 continued:

	w	x	y	z	f1	f2	f3
* (0,8,2,10)	–	0	–	0	0	0	1
* (2,10,3,11)	–	0	1	–	0	0	1
* (2,10,6,14)	–	–	1	0	0	0	1
* (8,9,10,11)	1	0	–	–	0	0	1
* (5,13,7,15)	–	1	–	1	1	0	0
* (12,14,13,15)	1	1	–	–	1	0	0

So MOPI's are: For f1, f2, f3 : $wx'y'z$, $wxyz'$

For f2, f3 : $x'y'z'$ For f1, f2 : $w'y'z$

For f1 : $wy'z'$, xz, wx

For f2 : $w'x'y'$

For f3 : $x'z'$, $x'y$, yz', wx'

Part (ii)

		f1								f2					f3								
		1	5	7	8	12	13	14	15	0	1	5	8	14	0	2	3	6	8	9	10	11	14
8	$wx'y'z$				X								X						X				
14	$wxyz'$							X						X									X
0,8	$x'y'z'$									X			X		X				X				
1,5	$w'y'z$	X	X								X	X											
8,12	$wy'z'$				X	X																	
5,7,13,15	xz		X	X			X		X														
12,13,14,15	wx					X	X	X	X														
0,1	$w'x'y'$									X	X												
0,2,8,10	$x'z'$														X	X			X		X		
2,3,10,11	$x'y$															X	X				X	X	
2,6,10,14	yz'															X		X			X		X
8,9,10,11	wx'																		X	X	X	X	

So $f1 = w'y'z + xz +$

$f2 = wxyz' + w'y'z +$

$f3 = x'y + yz' + wx' +$

Eliminate the columns covered by essential mopI's to get

	f1			f2		f3	Cg	Cd
	8	12	14	0	8	0		
8 $wx'y'z$	X				X		1	5,6
14 $wxyz'$			X				0	1
0,8 $x'y'z'$				X	X	X	1	4,5
8,12 $wy'z'$	X	X					1	4
12,13,14,15 wx		X	X				1	3
0,1 $w'x'y'$				X			1	4
0,2,8,10 $x'z'$						X	1	3

For f1, the combination for the lowest cost is (14) + (8, 12)

For f2, (0,8) has lower cost than (8) + (0,1)

For f3, Since (0,8) is chosen for f2, the gate cost for (0,8) becomes 0. So choose (0,8) over (0,2,8,10).

Therefore,

$f1 = w'y'z + xz + wxyz' + wy'z'$

$f2 = wxyz' + w'y'z + x'y'z'$

$f3 = x'y + yz' + wx' + x'y'z'$

3. Problem 5.6 (a)

$S_{1,3,5,7}(x_1,\ldots x_7)S_{4,5,6,7}(x_1,\ldots x_7)= S_{5,7}(x_1,\ldots x_7)$

Reason: When ANDing two symmetric functions, only the S_a terms that appear in both functions will remain. To illustrate this, consider the following (smaller) example:

$S_{1,2}(a,b,c)S_{2,3}(a,b,c)$.

Expand these out into regular functions:

$(ab'c' + a'bc' + a'b'c + abc' + ab'c + a'bc)(abc' + ab'c + a'bc + abc)$

Each term in the left portion, when multiplied out will cancel with every term in the right half, except if it is identical. For example $ab'c' * abc' = abb'c' = 0$. Therefore after this is multiplied out, only the following terms remain: $abc' + ab'c + a'bc = S_2(a,b,c)$

This works in general as well under similar logic.

Problem 5.6 (d)

$x_1'S_{0,1,2}(x_2,x_3,x_4) + x_1S_{2,3}(x_2',x_3',x_4') = S_{0,1,2}(x_1,x_2,x_3,x_4)$

To arrive at this, first express $S_{2,3}(x_2',x_3',x_4')$ as a function of only x_2,x_3,x_4:

$S_{2,3}(x_2',x_3',x_4') = S_{0,1}(x_2,x_3,x_4)$.

To see this, consider, only the $S_3(x_2',x_3',x_4')$ term. This will be 1 when each of its variables are 1, i.e. $x_2= x_3=x_4=0$. This is the same as $S_0(x_2,x_3,x_4)$.

Now the following changes can be made (expand them out if you don't understand the reasoning).

$x_1'S_0(x_2,x_3,x_4)=S_0(x_1,x_2,x_3,x_4)$
$x_1S_0(x_2,x_3,x_4)+x_1'S_1(x_2,x_3,x_4)=S_1(x_1,x_2,x_3,x_4)$
$x_1S_1(x_2,x_3,x_4)+x_1'S_2(x_2,x_3,x_4)=S_2(x_1,x_2,x_3,x_4)$

4. Problem 5.7 (e)
K-Map:

yz \ wx	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	1	0	0	0
10	1	0	0	0

$F(w,x,y,z)=w'x'+w'y'+x'y'$

W:

fw : x'y'

fw': x'+y'+x'y'

fw => fw' negative in w

X:

fx: w'y'

fx': w'+w'y'+y'

fx => fx' negative in x

Y:

fy: w'x'

fy': w'x'+w'+x'

fy => fy' negative in y

Z:

fz: w'x'+w'y'+x'y'

fz': w'x'+w'y'+x'y'

fz = fz' vacuous in z

Therefore it is unate, since it is negative in w,x,y and vacuous in z (which is trivially positive or negative)

Problem 5.7 (f) K-Map:

yz \ wx	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	1	0	0	0
10	1	0	0	0

F(w,x,y,z)=w'x'+w'y'

W:

fw: 0

fw': x'+y'

fw => fw' negative in w

X:

fx: w'y'

fx': w'+w'y'

fx => fx' negative in x

Y:

fy: w'x'

fy': w'x'+w'

fy => fy' negative in y

Z:

fz: w'x'+w'y'
fz': w'x'+w'y'

fz = fz' vacuous in z

Therefore it is unate (same reason as in previous problem).

5.
fxi * fxi' = fxi' means that whenever fxi' is 1, fxi * fxi' is also 1, and therefore fxi is also 1. By definition then fxi => fxi' (see Page 171 on text book). Thus by theorem 5.3-3 (Page 172 on text book) f is positive in xi.

6.

w	x	y	z	f	
0	0	0	0	0	$0<T$
0	0	0	1	0	$a_z<T$
0	0	1	0	0	$a_y<T$
0	0	1	1	0	$a_y + a_z<T$
0	1	0	0	0	$a_x<T$
0	1	0	1	0	$a_x+a_z<T$
0	1	1	0	0	$a_x+a_y<T$
0	1	1	1	0	$a_x+a_y+a_z<T$
1	0	0	0	0	$a_w<T$
1	0	0	1	0	$a_w+a_z<T$
1	0	1	0	0	$a_w+a_y<T$
1	0	1	1	1	$a_w+a_y+a_z>T$
1	1	0	0	1	$a_w+a_x>T$
1	1	0	1	1	$a_w+a_x+a_z>T$
1	1	1	0	1	$a_w+a_x+a_y>T$
1	1	1	1	1	$a_w+a_x+a_y+a_z>T$

One possible solution to satisfy all of these:

$a_w=5$
$a_x=2$
$a_y=1$
$a_z=1$
$T=6.5$

7. Prove f = (x' + y')(x + wy) is not a threshold function in two methods:

Method 1: prove threshold does not exit:

W	X	Y	f		
0	0	0	0		
0	0	1	0		
0	1	0	1	$a_x > T$	(A)
0	1	1	0	$a_x + a_y < T$	(B)
1	0	0	0	$a_w < T$	(C)
1	0	1	1	$a_w + a_y > T$	(D)
1	1	0	1		
1	1	1	0		

From condition (A) and (B), $a_y < 0$ (E)
(C) + (E) => $a_w + a_y < T$ ***contradicts*** (D)
Hence the threshold does not exist.

Method 2: Prove the function is not unate.

$f_x = f(w, x=1, y) = y'$
$f_{x'} = f(w, x=0, y) = wy$

$f_x * f_{x'} = 0 \neq f_x$ or $f_{x'}$. f_x and $f_{x'}$ are not compatible; f is mixed in x.
Hence f is not unate, and therefore not a threshold function.