ECE 462 Spring 2005 Homework 4 Solutions

1. 5.14

Prove $\frac{df}{dx_i} = f(x_1, x_2, ..., x_i, ..., x_n) \oplus f(x_1, x_2, ..., \overline{x}_i, ..., x_n) =$

$$\frac{df}{dx_i} = f(x_1, x_2, ..., x_i = 1, ..., x_n) \oplus f(x_1, x_2, ..., x_i = 0, ..., x_n)$$

$$\frac{df}{dx_i} = (x_i f(x_1, x_2, ..., x_i = 1, ..., x_n) + \overline{x}_i f(x_1, x_2, ..., x_i = 0, ..., x_n)) \oplus$$
$$(\overline{x}_i f(x_1, x_2, ..., x_i = 1, ..., x_n) + x_i f(x_1, x_2, ..., x_i = 0, ..., x_n))$$

$$= (x_i fx_i + \overline{x}_i f\overline{x}_i) \oplus (\overline{x}_i fx_i + x_i f\overline{x}_i)$$

The ORs can be replaced by XORs because the two parts of each function will only be 1 at the same time when the function is vacuous in x_i. If that were the case, then the two definitions would be trivially equivalent. Thus:

$= x_i fx_i \oplus \overline{x}_i f\overline{x}_i \oplus \overline{x}_i fx_i \oplus x_i f\overline{x}_i$ (combining equal residuals)

$$= (x_i \oplus \overline{x}_i) fx_i \oplus (\overline{x}_i \oplus x_i) f\overline{x}_i \Rightarrow fx_i \oplus x_i f\overline{x}_i$$

$$= f(x_1, x_2, ..., x_i = 1, ..., x_n) \oplus f(x_1, x_2, ..., x_i = 0, ..., x_n)$$

2. 5.15(a)

$$f(x,y,z) = (x'+y)(x+z)$$
$$f_{\overline{x}} = z$$
$$f_x = y$$
$$df/dx = y \oplus z$$
$$= yz' + y'z$$

$$f(x,y,z) = (x'+y)(x+z)$$
$$f_{\overline{y}} = x'(x+z) = x'z$$
$$f_y = x+z$$
$$df/dy = (x'z) \oplus (x+z)$$
$$= (x+z)(x'z)' + (x+z)'(x'z)$$
$$= (x+z)(x+z') + x'z'(x'z)$$
$$= x + xz' + xz + xz' + x'z'z$$
$$= x$$

$$f(x,y,z) = (x'+y)(x+z)$$
$$f_{\overline{z}} = (x'+y)x = xy$$
$$f_z = x'+y$$
$$df/dz = (xy) \oplus (x'+y)$$
$$= xy(x'+y)' + (xy)'(x'+y)$$
$$= xy(xy') + (x'+y')(x'+y)$$
$$= x'$$

5.15(b)

$f(x,y,z) = x'y'(x'+yz) = x'y'+x'y'yz = x'y$

$f_x = 0, f_{\bar{x}} = y$

$df/dx = 0 \oplus y = \bar{y}$

$f_y = \bar{x}, f_{\bar{y}} = 0$

$df/dy = \bar{x} \oplus 0 = \bar{x}$

$f_z = \bar{x}y, f_{\bar{z}} = \bar{x}y$

$df/dz = \bar{x}y \oplus \bar{x}y = 0$

3. 5.16(a)

$f(w,x,y,z) = \sum(0,4,5,7,9,10,15)$

yz \ wx	00	01	11	10
00	1	1	0	0
01	0	1	0	1
11	0	1	1	0
10	0	0	0	1

$f_{y'}$

z \ wx	00	01	11	10
0	1	1	0	0
1	0	1	0	1

f_y

z \ wx	00	01	11	10
0	0	0	0	1
1	0	1	1	0

$f_{z'}$

y \ wx	00	01	11	10
0	1	1	0	0
1	0	0	0	1

f_z

y \ wx	00	01	11	10
0	0	1	0	1
1	0	1	1	0

yz \ x	0	1
00	1	1
01	0	1
11	0	1
10	0	0

$f_{w'}$

yz \ x	0	1
00	0	0
01	1	0
11	0	1
10	1	0

f_w

yz \ w	0	1
00	1	0
01	0	1
11	0	0
10	0	1

$f_{x'}$

yz \ w	0	1
00	1	0
01	1	0
11	1	1
10	0	0

f_x

By inspecting each of the residues, it's observed that there are no simple disjoint compositions for $f(g(w,x,z),y)$, $f(g(w,x,y),z)$, $f(g(w,y,z),x)$,or $f(g(x,y,z),w)$.

yz \ wx	00	01	11	10
00	1	1	0	0
01	0	1	0	1
11	0	1	1	0
10	0	0	0	1

By inspecting the k-map, it's possible to see:
No simple disjoint composition exists for
$f(g(y,z),w,x)$ or $f(g(w,x),y,z)$

xz \ wy	00	01	11	10
00	1	0	1	0
01	0	0	0	1
11	1	1	1	0
10	1	0	0	0

By inspecting the k-map, it's possible to see:
No simple disjoint composition exists for
$f(g(x,z),w,y)$ or $f(g(w,y),x,z)$

xy \ wz	00	01	11	10
00	1	0	1	0
01	0	0	0	1
11	0	1	1	0
10	1	1	0	0

By inspecting the k-map, it's possible to see:
No simple disjoint composition exists for
$f(g(w,z),x,y)$ or $f(g(x,y),w,z)$

5.16(b)

$f(w,x,y,z) = \sum(0,3,4,11,12,15)$

yz \ wx	00	01	11	10
00	1	1	1	0
01	0	0	0	0
11	1	0	1	1
10	0	0	0	0

$f_{y'}$

z \ wx	00	01	11	10
0	1	1	1	0
1	0	0	0	0

f_y

z \ wx	00	01	11	10
0	1	0	0	0
1	1	0	1	1

$f_{z'}$

y \ wx	00	01	11	10
0	1	1	1	0
1	1	0	0	0

f_z

y \ wx	00	01	11	10
0	0	0	0	0
1	1	0	1	1

yz \ x	0	1
00	1	1
01	0	0
11	1	0
10	0	0

$f_{w'}$

yz \ x	0	1
00	0	1
01	0	0
11	1	1
10	0	0

f_w

yz \ w	0	1
00	1	0
01	0	0
11	1	1
10	0	0

$f_{x'}$

yz \ w	0	1
00	1	1
01	0	0
11	0	1
10	0	0

f_x

By inspecting each of the residues, it's observed that there are no simple disjoint compositions for $f(g(w,x,z),y)$, $f(g(w,x,y),z)$, $f(g(w,y,z),x)$,or $f(g(x,y,z),w)$.

yz \ wx	00	01	11	10
00	1	1	1	0
01	0	0	0	0
11	1	0	1	1
10	0	0	0	0

By inspecting the k-map, it's possible to see:
No simple disjoint composition exists for $f(g(y,z),w,x)$ or $f(g(w,x),y,z)$

xz \ wy	00	01	11	10
00	1	0	0	0
01	0	1	1	0
11	0	0	1	0
10	1	0	0	1

By inspecting the k-map, it's possible to see:
No simple disjoint composition exists for $f(g(x,z),w,y)$ or $f(g(w,y),x,z)$

xy \ wz	00	01	11	10
00	1	0	0	0
01	0	1	1	0
11	0	0	1	0
10	1	0	0	1

By inspecting the k-map, it's possible to see:
No simple disjoint composition exists for $f(g(w,z),x,y)$ or $f(g(x,y),w,z)$

4. 5.18(a) Find all simple disjoint compositions of the form $F(a,b,c,g(d,e))$ for the function $f(v,w,x,y,z)=\sum(0,2,3,12,14,15,17,20,21,22,23,24,25,26,27,29)$.

v=0:

yz \ wx	00	01	11	10
00	1	0	1	0
01	0	0	0	0
11	1	0	1	0
10	1	0	1	0

v=1:

yz \ wx	00	01	11	10
00	0	1	0	1
01	1	1	1	1
11	0	1	0	1
10	0	1	0	1

yz	$f_{w'x'v'}$	$f_{w'xv'}$	$f_{wxv'}$	$f_{wx'v'}$	$f_{w'x'v}$	$f_{w'xv}$	f_{wxv}	$f_{wx'v}$
00	1	0	1	0	0	1	0	1
01	0	0	0	0	1	1	1	1
11	1	0	1	0	0	1	0	1
10	1	0	1	0	0	1	0	1

By observation, a simple disjoint composition exists for $f(w,x,v,g(y,z))$ of the form $f(w,x,v,y'z)$

wx	00	01	11	10
$f_{y'z'v'}$	1	0	1	0
$f_{y'zv'}$	0	0	0	0
$f_{yzv'}$	1	0	1	0
$f_{yz'v'}$	1	0	1	0

wx	00	01	11	10
$f_{y'z'v}$	0	1	0	1
$f_{y'zv}$	1	1	1	1
f_{yzv}	0	1	0	1
$f_{yz'v}$	0	1	0	1

By observation, a simple disjoint composition exists for $f(v,y,z,g(w,x))$ of the form $f(v,y,z,w'x+wx')$.

v=0:

xz \ wy	00	01	11	10
00	1	1	0	0
01	0	1	0	0
11	0	0	1	0
10	0	0	1	1

v=1:

xz \ wy	00	01	11	10
00	0	0	1	1
01	1	0	1	1
11	1	1	0	1
10	1	1	0	0

xz	$f_{w'y'v'}$	$f_{w'yv'}$	$f_{wyv'}$	$f_{wy'v'}$	$f_{w'y'v}$	$f_{w'yv}$	f_{wyv}	$f_{wy'v}$
00	1	1	0	0	0	0	1	1
01	0	1	0	0	1	0	1	1
11	0	0	1	0	1	1	0	1
10	0	0	1	1	1	1	0	0

By observation, no simple disjoint composition exists for $f(w,y,v,g(x,z))$

wy	00	01	11	10
$f_{x'z'v'}$	1	1	0	0
$f_{x'zv'}$	0	1	0	0
$f_{xzv'}$	0	0	1	0
$f_{xz'v'}$	0	0	1	1

wy	00	01	11	10
$f_{x'z'v}$	0	0	1	1
$f_{x'zv}$	1	0	1	1
f_{xzv}	1	1	0	1
$f_{xz'v}$	1	1	0	0

By observation, no simple disjoint composition exists for $f(x,z,v,g(w,y))$.

xy \ wz	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	0	0	1	1
10	0	0	0	1

v=0

xy \ wz	00	01	11	10
00	0	1	1	1
01	0	0	1	1
11	1	1	0	0
10	1	1	1	0

v=1

xy	$f_{w'z'v'}$	$f_{w'zv'}$	$f_{wzv'}$	$f_{wz'v'}$	$f_{w'z'v}$	$f_{w'zv}$	f_{wzv}	$f_{wz'v}$
00	1	0	0	0	0	1	1	1
01	1	1	0	0	0	0	1	1
11	0	0	1	1	1	1	0	0
10	0	0	0	1	1	1	1	0

By observation, no simple disjoint composition exists for $f(w,z,v,g(x,z))$.

wz	00	01	11	10
$f_{x'y'v'}$	1	0	0	0
$f_{x'yv'}$	1	1	0	0
$f_{xyv'}$	0	0	1	1
$f_{xy'v'}$	0	0	0	1

wz	00	01	11	10
$f_{x'y'v}$	0	1	1	1
$f_{x'yv}$	0	0	1	1
f_{xyv}	1	1	0	0
$f_{xy'v}$	1	1	1	0

By observation, no simple disjoint composition exists for $f(x,y,v,g(w,z))$.

More residues to consider:

yz	$f_{w'x'y'}$	$f_{w'x'y}$	$f_{w'xy'}$	$f_{w'xy}$	$f_{wxy'}$	f_{wxy}	$f_{wx'y'}$	$f_{wx'y}$
00	1	1	0	0	1	1	0	0
01	0	1	0	0	0	1	0	0
11	0	0	1	1	0	0	1	1
10	1	0	1	1	1	0	1	1

By observation, no simple disjoint composition exists for $f(w,x,y,g(v,z))$.

vy	$f_{w'x'z'}$	$f_{w'x'z}$	$f_{w'xz'}$	$f_{w'xz}$	$f_{wxz'}$	f_{wxz}	$f_{wx'z'}$	$f_{wx'z}$
00	1	0	0	0	1	0	0	0
01	1	1	0	0	1	1	0	0
11	0	0	1	1	0	0	1	1
10	0	1	1	1	0	1	1	1

By observation, no simple disjoint composition exists for $f(w,x,z,g(v,y))$.

vx	$f_{w'y'z'}$	$f_{w'y'z}$	$f_{w'yz'}$	$f_{w'yz'}$	$f_{wyz'}$	f_{wyz}	$f_{wy'z'}$	$f_{wy'z}$
00	1	0	1	0	0	1	0	0
01	0	0	0	1	1	0	1	0
11	0	1	0	1	1	0	1	1
10	1	1	1	0	0	1	0	1

By observation, no simple disjoint composition exists for $f(w,y,z,g(v,x))$.

vw	$f_{y'z'x'}$	$f_{y'z'x}$	$f_{y'zx'}$	$f_{y'zx}$	$f_{yzx'}$	f_{yzx}	$f_{yz'x'}$	$f_{yz'x}$
00	1	1	0	0	1	1	1	1
01	0	0	0	0	0	0	0	0
11	0	0	1	1	0	0	0	0
10	1	1	1	1	1	1	1	1

By observation, no simple disjoint composition exists for $f(y,z,x,g(v,w))$.

So, two simple disjoint decompositions exist for function f: $f(v,y,z,w'x+wx')$ and $f(w,x,v,y'z)$.

5. Problem 3.8(a)

$f(a,b,c,d) = (((a'b')'c(ad)')'(a'cd')')'$

$= (((a'b')'c(ad)') + (a'cd')$

$= (a+b)c(a'+d') + a'cd'$

$= (ac+bc)(a'+d') + a'cd'$

$= aa'c + a'bc + acd' + bcd' + a'cd'$

1 sets: $[a,a',c],[a',b,c],[a,c,d'],[b,c,d'],[a'c,d']$

① ② ③ ④ ⑤

(b)

ab \ cd	00	01	11	10
00	0	0	0	1
01	0	0	1	1
11	0	0	0	1
10	0	0	0	1

By inspection, a static one hazard exists <a,b,c,d>: <0010> - <1010>

By inspection, a static zero hazard exists <a,b,c,d>: <0011>-<1011>

(c)

$f(a,b,c,d) = (a+b)c(a'+d') + a'cd'$

$f^D = (ab + c + a'd')(a'+c+d')$

$= aa'b + abc + abd' + a'c + c + cd' + a'd' + a'cd + a'd'$

$= aa'b + abd' + c + a'd'$

$(f^D)^D = (a + a' + b)(a + b + d')(c)(a' + d')$

0-sets: [a, a', b], [a, b, d'], [c], [a', d']

(d)

ab \ cd	00	01	11	10
00	0	0	0	1
01	0	0	1	1
11	0	0	0	1
10	0	0	0	1

By inspection, a static one hazard exists <a,b,c,d>: <0010> - <1010>

By inspection, a static zero hazard exists <a,b,c,d>: <0011>-<1011>

6.circuit from 3.11(a)

$f = ((BD)'(((AB)'AC)'(AB)'C)')'$

$= (BD) + (AB + A'+C')(A'+B')C$

$= BD + (AA'B + A'+A'C'+ABB'+A'B'+B'C')C$

$= BD + AA'BC + A'C + A'CC'+ABB'C + B'C'C$

$= BD + A'C + ABB'C + B'CC'$

1-sets: [B, D], [A’,C], [A,B,B’,C], [B’,C,C’]
(1) (2) (3) (4)

$f = (BD) + (AB + A'+C')(A'+B')C$

$f^D = (B + D)((A + B)A'C'+A'B'+C)$

$= (B + D)(AA'C'+A'BC''+A'B'+C$

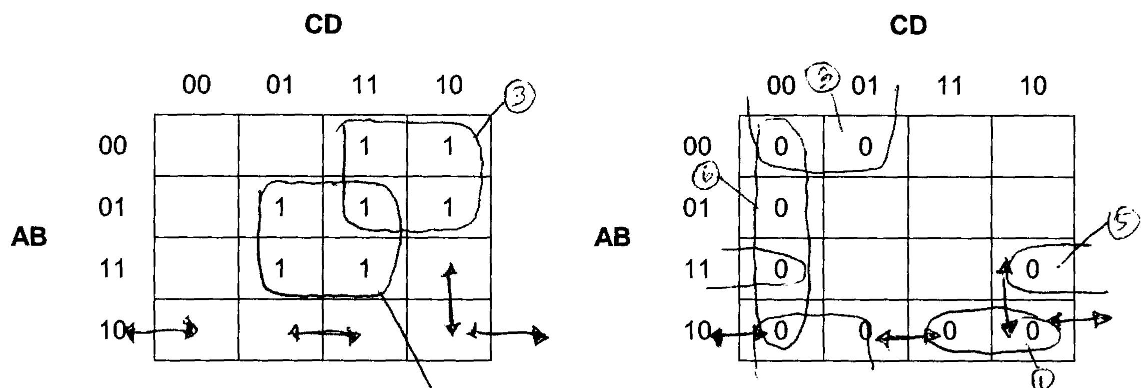
$= AA'BC'+A'BC'+A'BB'+BC + AA'C'D + A'BC'D + A'B'D + CD$

$= A'BC'+A'BB'+BC + AA'C'D + A'B'D + CD$

$(f^D)^D = (A'+B + C')(A'+B + B')(B + C)(A + A'+C'+D)(A'+B'+D)(C + D)$

0-sets: [A’,B,C’], [A’,B,B’], [B,C], [A,A’,C’,D], [A’,B’,D], [C,D]
(1) (2) (3) (4) (5) (6)

(ii)



active unstable 1-sets between two zero’s.

transitions between two zeros not covered by a single stable zero set.

(iii)

By inspection, static-zero hazards exist during transitions:

abcd: <1001> - <1011>

<1110> - <1010>

<1010> - <1000>

No static-one hazards exist.