ECE 462 Homework 5 Solutions

1. From Homework 5, the static-0 hazards exist for transitions

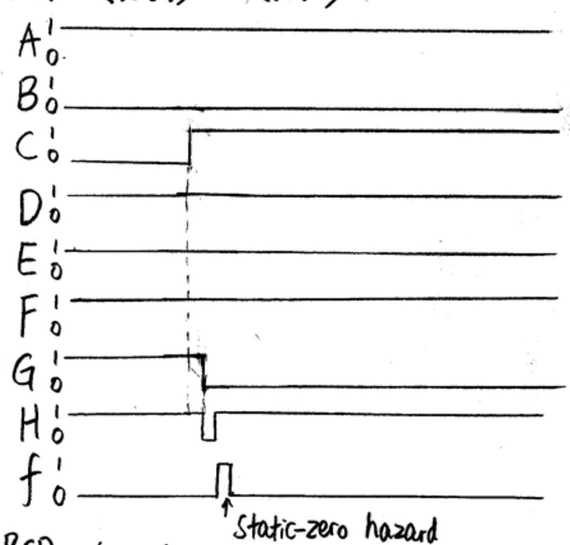
ABCD: ⟨1001⟩ — ⟨1011⟩

⟨1110⟩ — ⟨1010⟩

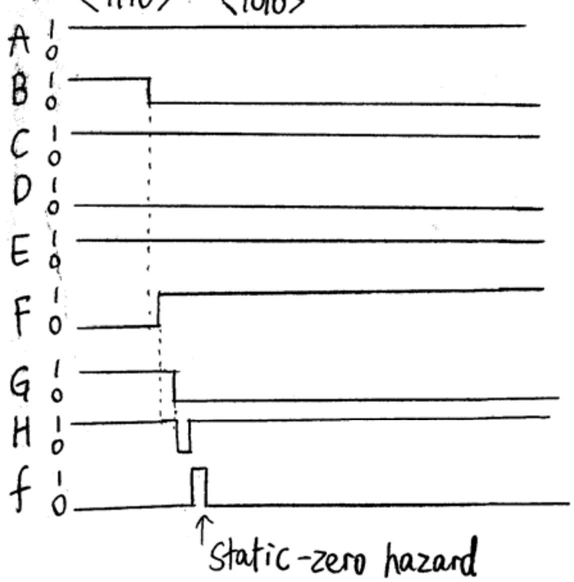
⟨1000⟩ — ⟨1010⟩

plot the timing diagrams as following:

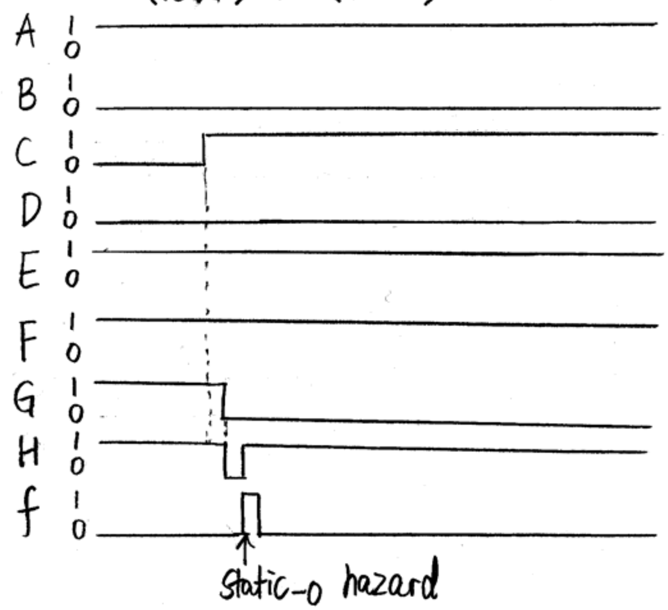
ABCD ⟨1001⟩ ← ⟨1011⟩



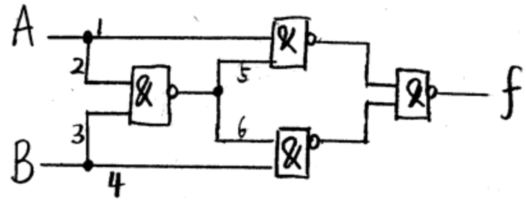
ABCD ⟨1110⟩ ← ⟨1010⟩



ABCD $\langle 1000 \rangle \rightarrow \langle 1010 \rangle$



2.



(i) $f(A,B) = [(A_1(A_{2,5}B_{3,5})')'(B_4(A_{2,6}B_{3,6})')']'$

(Let $A_{2,5} = A_7$, $A_{2,6} = A_8$, $B_{3,5} = B_7$, $B_{3,6} = B_8$)

$= (A_1(A_7B_7)') + (B_4(A_8B_8)')$

$= A_1(A_7' + B_7') + B_4(A_8' + B_8')$

$= A_1A_7' + A_1B_7' + A_8'B_4 + B_4B_8'$

P-sets : $[A_1, A_7']$, $[A_1, B_7']$, $[A_8', B_4]$, $[B_4, B_8']$

$f = A_1(A_7' + B_7') + B_4(A_8' + B_8')$

$f^D = (A_1 + A_7'B_7')(B_4 + A_8'B_8')$

continue f^D:

$$= A_1 B_4 + A_1 A_8' B_8' + A_7' B_7' B_4 + A_7' A_8' B_7' B_8'$$

$$f = (f^D)^D = (A_1 + B_4)(A_1 + A_8' + B_8')(A_7' + B_7' + B_4)(A_7' + A_8' + B_7' + B_8')$$

S-sets: $[A_1, B_4]$, $[A_1, A_8', B_8']$, $[A_7', B_7', B_4]$, $[A_7', A_8', B_7', B_8']$

(ii) Unstable P-sets: $[A_1, A_7']$ $[B_4, B_8']$

Stable P-sets: $[A_1, B_7']$ $[A_8', B_4]$

$\leftarrow$ Use the steps corresponding to the procedures listed on Page 91

1) Let $U_1 = [A_1, A_7']$, $X_1 = A\{1, 7\}$

2) Let $B_1 = [A_8', B_4]$

3) $B = 1$

4) No work

$\Rightarrow$ $\boxed{\text{Dynamic hazard } \langle A, B\rangle : \langle 0,1\rangle - \langle 1,1\rangle \text{ (A changing, B=1)}}$

1) Let $U_2 = [B_4, B_8']$, $X_2 = B\{4, 8\}$

2) Let $B_2 = [A_1, B_7']$

3) $A = 1$

4) No work

$\Rightarrow$ $\boxed{\text{Dynamic hazard } \langle A, B\rangle : \langle 1,0\rangle - \langle 1,1\rangle \text{ (A=1, B changing)}}$

(iii) Unstable S-sets: $[A_1, A_8', B_8']$, $[A_7', B_7', B_4]$

Stable S-sets: $[A_1, B_4]$, $[A_7', A_8', B_7', B_8']$

1) Let $U_1 = [A_1, A_8', B_8]$, $X_1 = A\{1, 8\}$

2) Let $B_1 = [A_7', A_8', B_7', B_8']$

3) $B = 1$

4) No work

$\Rightarrow$ $\boxed{\text{Dynamic hazard } \langle A, B\rangle : \langle 0,1\rangle - \langle 1,1\rangle \text{ (A changing, B=1)}}$

1) Let $U_2 = [A_1', B_1', B_4]$, $X_2 = B\{7, 4\}$

2) Let $B_2 = [A_1', A_8', B_1', B_8']$

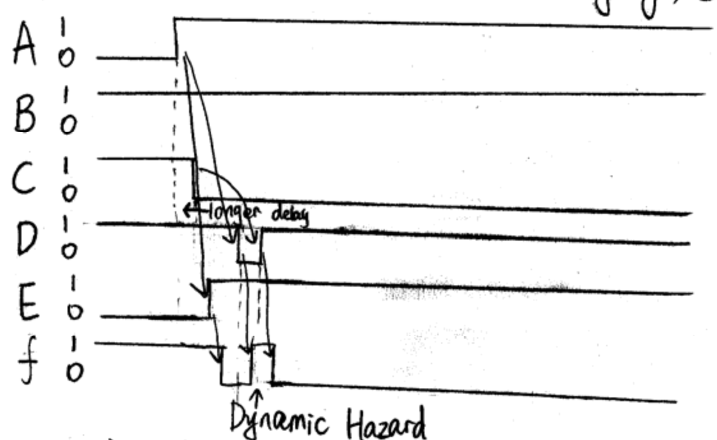
3) $A=1$

4) No work

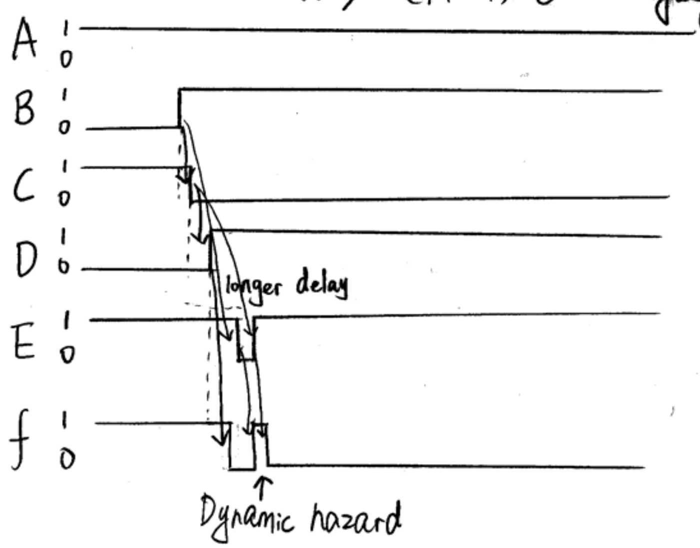
$\Rightarrow$ $\boxed{\text{Dynamic hazard: } \langle A, B\rangle : \langle 10\rangle - \langle 11\rangle \text{ (or } A=1,\ B \text{ changing)}}$

3. Plot the two dynamic hazards discovered in Problem 2:

$\langle A, B\rangle : \langle 01\rangle - \langle 11\rangle$ (or A-changing, $B=1$)

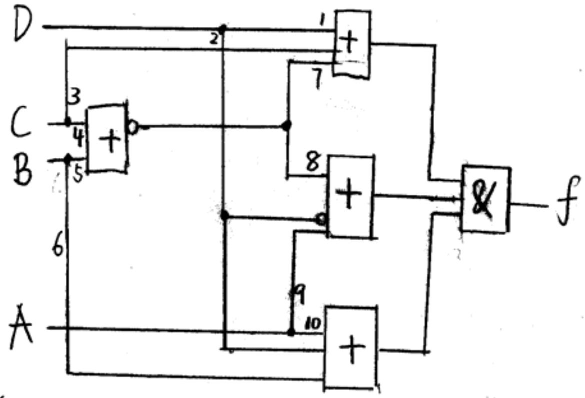


$\langle A, B\rangle : \langle 10\rangle - \langle 11\rangle$ ($A=1$, B changing)



4. Problem 3.9

Mark the Logic Diagram as following:



(a) $f = (D1 + C3 + (C4 + B5)'_7)((C_4 + B5)'_8 + D_2' + A_9)(A_{10} + D_2 + B_6)$

$f^D = D1\,C3(C_{4,7}\,B_{5,7})' + (C_{4,8}\,B_{5,8})'\,D_2'A_9 + A_{10}D_2B_6$

(Let $C_{4,7} = C_{11}$, $C_{4,8} = C_{12}$, $B_{5,7} = B_{11}$, $B_{5,8} = B_{12}$)

$= D_1C_3(C_{11}' + B_{11}') + (C_{12}' + B_{12}')D_2'A_9 + A_{10}D_2B_6$

$= D_1C_3C_{11}' + D_1C_3B_{11}' + C_{12}'D_2'A_9 + B_{12}'D_2'A_9 + A_{10}D_2B_6$

$f = f^{DD} = (D_1 + C_3 + C_{11}')(D_1 + C_3 + B_{11}')(C_{12}' + D_2' + A_9)(B_{12}' + D_2' + A_9)(A_{10} + D_2 + B_6)$

So the S-Sets: $\underbrace{[D_1, C_3, C_{11}']}_{\text{unstable}}, \underbrace{[D_1, C_3, B_{11}'], [C_{12}', D_2', A_9], [B_{12}', D_2', A_9], [A_{10}, D_2, B_6]}_{\text{stable}}$

(b)

AB \ CD	00	01	11	10
00	0	1	0	0
01	0	0	0	1
11	0	1	1	1
10	1	1	1	1

K-Map with S-sets

continue 4

(c)

$U = [D_1, C_3, C_{11}']$, $X = c$

$B = [C_{12}', D_2', A_9]$

In order to make U active, $D = 0$
In order to make B active, $D = 1$, $A = 0$ } conflicts with each other

Hence it is impossible to make U and B active at the same time

There are no other possible combination of U and B.

Therefore, no dynamic hazard exists.

5. The K-Map of the Logic circuit is as following:

AB	00	01	11	10
00	0_0	0_1	1_3	1_2
01	0_4	1_5	1_7	1_6
11	0_{12}	1_{13}	1_{15}	0_{14}
10	0_8	0_9	0_{11}	0_{10}

The prime implicants are BD (5,7,13,15) and $\bar{A}C$ (2,3,6,7)

Use the tabular method to determine hazard-free minimal sum

	(2,3)	(2,6)	(3,7)	(5,7)	(6,7)	(5,13)	(7,15)	(13,15)
BD (5,7,13,15)				X		X	X	X
$\bar{A}C$ (2,3,6,7)	X	X	X		X			

The table shows that all the 1 pairs are covered by the prime implicants. Hence the hazard-free minimal sum is:

$$f = BD + \bar{A}C$$