

ECE 462 Logic Design (Spring 2005)

Total Points: 25

Duration: 50 minutes

You may bring 1 double-sided sheet of handwritten notes to this test.

If you need to make additional assumptions to answer a question, please state your assumptions clearly.

Name: Solutions

This test is worth 23% of the final grade.

The test is printed on 6 pages.

1. (4 points) For the functions  $f(w,x,y,z) = \Sigma (0,1,5,7,13)$ , and  $g(w,x,y,z) = \Sigma (5,9,13)$ , using Quine-McCluskey tabulation procedure, determine all the multiple-output prime implicants.

Determine which of the MOPIs are implicants of function  $g$  but not implicants of function  $f$ .

You may use the table below if you like.

	Identifier	Tag		
		f	g	
0	0 0 0 0	1	0	✓
1	0 0 0 1	1	0	✓
5	0 1 0 1	1	1	✓
9	1 0 0 1	0	1	✓
7	0 1 1 1	1	0	✓
13	1 1 0 1	1	1	✓

\* MOPI's

* (0,1)	0 0 0 -	1 0	$\bar{w} \bar{x} \bar{y}$
* (1,5)	0 - 0 1	1 0	$\bar{w} \bar{y} z$
<del>(1,9)</del>	<del>- 0 0 1</del>	<del>0 0</del>	
* (5,7)	0 1 - 1	1 0	$\bar{w} x z$
* (5,13)	- 1 0 1	1 1	$x \bar{y} z$
* (9,13)	1 - 0 1	0 1	$w \bar{y} z$
<del>(1,5,9,13)</del>	<del>- - 0 1</del>	<del>0 0</del>	

WX

yz	00	01	11	10
00	1	0	0	0
01	1	1	1	0
11	0	1	0	0
10	0	0	0	0

f

WX

yz	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	0	0	0	0
10	0	0	0	0

g

WX

yz	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	0	0	0
10	0	0	0	0

f-g

2. (4 points) Consider function  $f(w,x,y,z) = \Sigma (0,3,5,7,8,10,11,12,13)$ . The prime implicant table for function  $f(w,x,y,z)$  is shown below. This function has eight prime implicants, labelled A through H in the prime implicant table.

(i) Identify all distinguished columns and essential rows in the prime implicant table.

(ii) Using the prime implicant method, find all minimal sums for function  $f(w,x,y,z)$ . Show your work.

You may denote each minimal sum as a sum of the letters (A through H) representing the desired prime implicants.

*essential row*

*distinguishing column*

	0	3	5	7	8	10	11	12	13
<i>*</i> A = (0,8)	X				X				
<i>**</i> B = (3,7)		X		X					
<i>CCD</i> C = (5,7)			X	X					
<i>**</i> D = (5,13)			X						X
E = (8,12)					X			X	
<i>FCG</i> F = (8,10)					X	X			
<i>**</i> G = (10,11)						X	X		
H = (12,13)								X	X
<i>**</i> I = (3,11)		X					X		

*Secondary essential*

Minimal Sum =  $A + G + B + D + E$

or  $A + G + B + D + H$

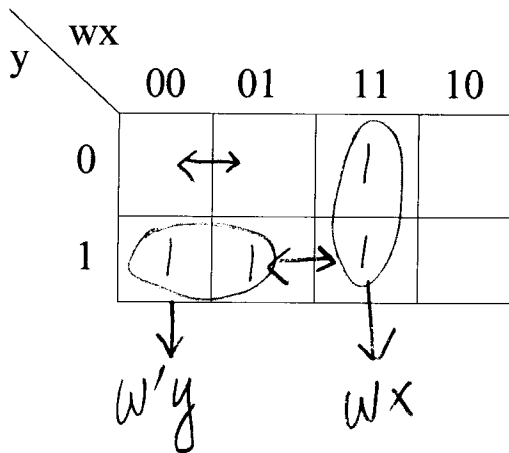
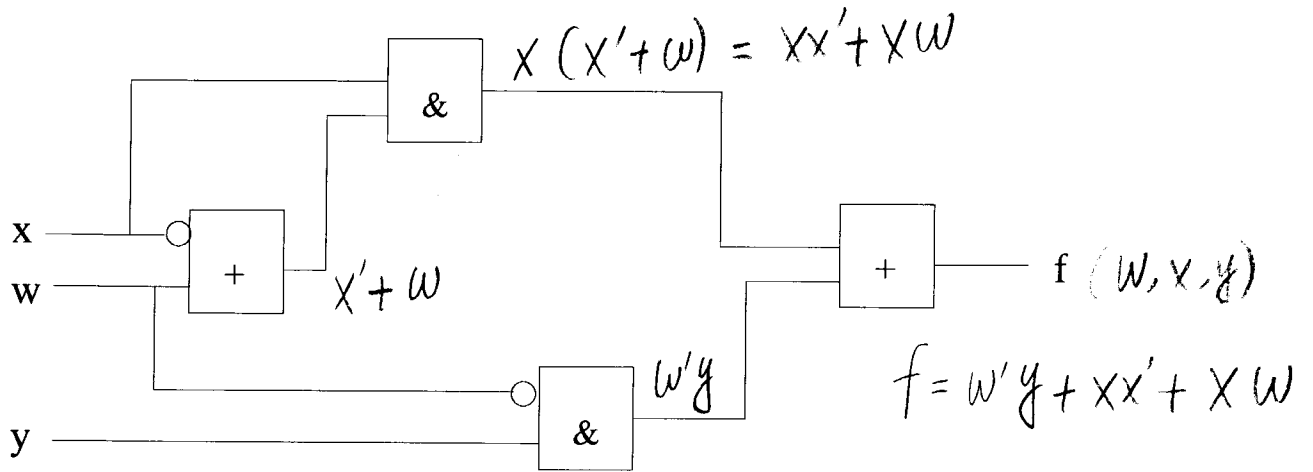
or  $A + G + B + C + H$

or  $A + G + H + C + I$

or  $A + C + F + H + I$

(Any one minimal sum is an adequate answer)

3. (4 points) Determine all the static logic hazards in the circuit shown below. Show your work.



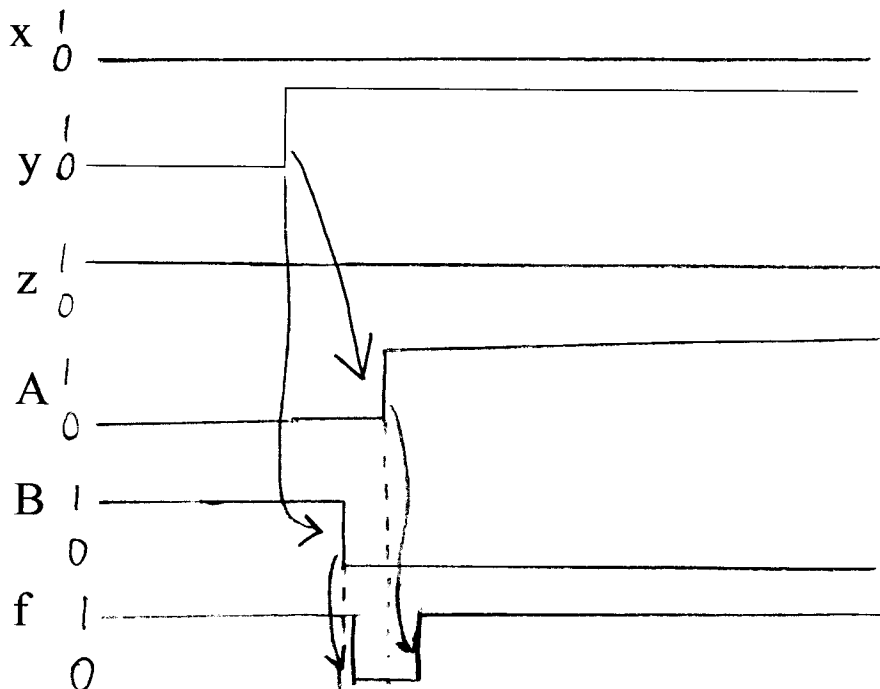
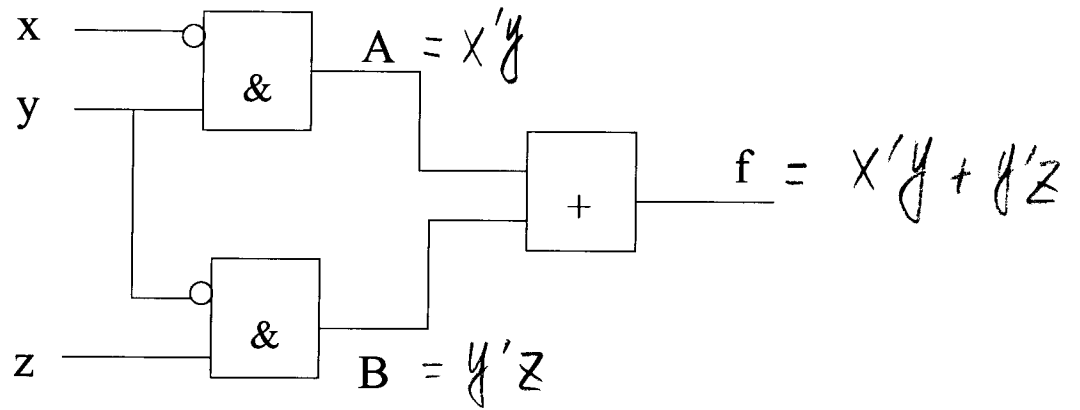
logic static 1 hazard

$$\langle wx y \rangle = 011 \leftrightarrow 111$$

logic static 0 hazard

$$\langle wx y \rangle = 000 \leftrightarrow 010$$

4. (4 points) The circuit shown below implements the function  $f(x,y,z) = x'y + y'z$ . There exists a static logic hazard for the transition from  $xyz = 001$  to  $xyz = 011$ . Complete the timing diagram below to show how this hazard may occur. Outputs of the AND gates in the circuit are labelled A and B as shown in the figure.



5. (5 points) Karnaugh map for a function  $f(w,x,y,z)$  is shown below.

		wx			
	yz	00	01	11	10
00		1	0	0	1
01		0	0	1	0
11		0	0	1	0
10		0	0	1	0

(a) For the above function, determine whether there exists a simple disjoint decomposition of the form  $F(g(y,z), w, x)$ . If such a decomposition exists, determine the appropriate functions  $g$  and  $F$  used for the decomposition.

Yes.  $g(y, z) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = y'z'$

$$F(g, w, x) = w'x'g + wxg' + wx'g = x'g + wxg'$$

Alternatively,  $g(y, z) = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} = y + z$

$$F(g, w, x) = w'x'g' + wxg + wx'g'$$

(b) Determine whether the function is positive or negative or mixed in variable  $w$ .

$f_{w'} \rightarrow$

1	0
0	0
0	0
0	0

$f_w \rightarrow$

1	0
0	1
0	1
0	1

Since  $f_{w'} \Rightarrow f_w$ ,  $f$  is positive

6. (4 points) Use the first Tison method to determine all the prime implicants for the function  $g(w,x,y) = y' + w'y + wx'y$

$$L = \{y', w'y, wx'y\}$$

$$\textcircled{w} \begin{matrix} w'y \\ wx'y \end{matrix} > \Rightarrow \text{CONSENSUS } x'y$$

$$\Rightarrow L = \{y', w'y, \cancel{wx'y}, x'y\}$$

$\textcircled{x}$  no consensus

$$\textcircled{y} \begin{matrix} y' \\ w'y \end{matrix} > \text{CONSENSUS } w'$$

$$\begin{matrix} y' \\ x'y \end{matrix} > \text{CONSENSUS } x'$$

$$L = \{y', \cancel{w'y}, \cancel{x'y}, w', x'\}$$

$$\Downarrow$$

$$L = \{y', w', x'\}$$

$\downarrow$   
P.I.'s

Alternate Method

$$\textcircled{y} \begin{matrix} y' \\ w'y \end{matrix} > \text{CONSENSUS } w'$$

$$\begin{matrix} y' \\ wx'y \end{matrix} > \text{CONSENSUS } wx'$$

$$L = \{y', \cancel{w'y}, \cancel{wx'y}, w', wx'\}$$

$$= \{y', w', wx'\}$$

$$\textcircled{w} \begin{matrix} w' \\ wx' \end{matrix} > \text{CONSENSUS } x'$$

$$L = \{y', w', \cancel{wx'}, x'\}$$

$$= \{y', w', x'\}$$

$\textcircled{x}$  no consensus

$$L = \{y', w', x'\}$$

$\downarrow$   
P.I.'s