CS/ECE 541 Homework # 3, Due in class September 15

1. A random walk on a ring is a stochastic process where there are N states, numbered 0 to $N-1$. Each step, a walker in state i steps to state $(i+1) \bmod N$ with probability p, and to state $(i-1) \bmod N$ with probability $1-p$ (take $-1 \bmod N = (N-1)$).

   - Assume $N = 4$. Write the transition probability matrix P for this process.
   - Assume that at time 0 the walk is in state 0, and that $p = 0.75$. Write down the state probability occupancy vector for the walk after 8 steps, and after 64 steps.
   - Prove that the under the assumptions above, $\pi = (0.25, 0.25, 0.25, 0.25)$.

2. A model of a communication channel assumes that if the last byte transmitted was in error, then the probability that the next byte is also in error is p_e. On the other hand, if the last byte was transmitted successfully, then the probability that the next byte is also transmitted successfully is p_s. Build a DTMC to determine the long-term fraction of bytes that are transmitted successfully, and give a closed form expression for that fraction.

3. Let's make the problem above a little more interesting. Suppose that a *message* consists of 4 bytes, and that for a message to be successfully received, all 4 bytes of it need to be correctly received. Suppose further that there is no feedback control, so always all 4 bytes of a message are transmitted, even if one or more of them are corrupted.

   - Build a DTMC that describes this process, and which could in principle be used to determine the fraction of messages that are transmitted successfully.
   - Assume that $p_e = 0.75$ and that $p_s = 0.999$, and compute the fraction of messages that are transmitted successfully.

4. Draw the state diagram of a DTMC that is reducible and periodic with all strongly connected components having period 2.