

## CS/ECE 541 Homework # 5, Due in class October 8, 2009

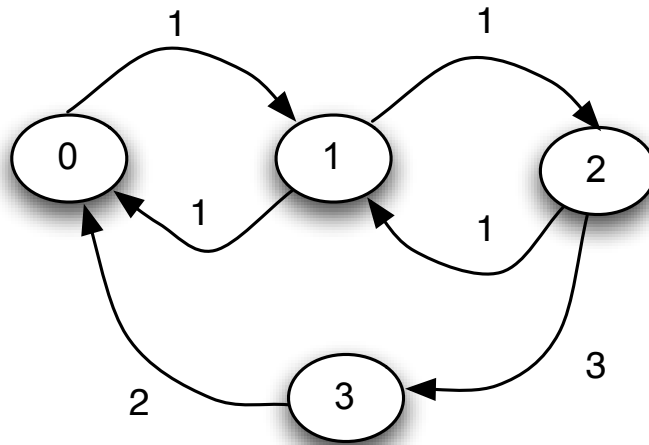
1. A “wash-by-hand” car wash employs four workers. The time required by a worker to wash a car is exponentially distributed with rate 0.1 minutes. Cars arrive to the car wash at rate 0.2 per minute (which is to say the the time between successive arrivals is exponentially distributed with rate 0.2). If a car arrives and there is an available worker, the car begins immediately to be washed by that worker. If a car arrives and all available workers are busy, then the car will wait for an available worker *unless* there is already a car waiting. In that case the car leaves. Thus at most one car is waiting at any given time.

When a worker finishes a car, if there is no car waiting for service, then with probability 0.1 he slips behind the building for a smoke, which takes an exponentially distributed amount of time, with rate 0.25 minutes. Any worker behind the building smoking is not considered to be available in the event that a car approaches the car wash for service.

Develop a CTMC of this system:

- What are the states and what do they mean?
- How would you go about deriving the probability that when a car approaches the car wash, it will eventually be serviced?
- How would you go about computing the fraction of time that *all workers* are behind the building for a smoke?
- How would you go about deriving the revenue lost by the owner due to cars arriving when an otherwise available worker is behind the building smoking?

2. Consider the CTMC illustrated below.



- What is the transition matrix for the DTMC based on uniformizing this chain with  $\lambda = 4$ ?
- Use uniformization to compute the state occupancy probability vector at time 32, given  $\pi_0 = (1, 0, 0, 0)$ .
- What is the asymptotic state occupancy probability vector for the uniformized DTMC?
- What do you think (in general) the relationship is between the asymptotic state occupancy probability vector for an CTMC, and the asymptotic state occupancy probability vector for the DTMC derived from uniformization? Why?