

Probability distribution of interest is  $\{p_i\} = \Pr\{X = i\}$ . Assume there are  $N$  values  $i$  for which  $p_i > 0$ . Let  $i \in S$  if  $p_i > 0$ . Each alias table entry of the form  $(v_j, u_j, s_j)$ , where  $v_j, u_j \in S$  and  $s_j$  is the probability of selecting  $v_j$  when this table entry is chosen.

1. **Initialization** Set  $q_i = Np_i$ , for all  $i \in S$ . Define  $G = \{i | i \in S \text{ and } q_i \geq 1\}$ , and  $H = \{i | i \in S \text{ and } q_i < 1\}$ .

2. **Build Table**

For  $j = 1$  to  $N$

if( $H \neq \emptyset$ ) {

REMOVE any  $i \in H$

set  $v_j = i$

set  $s_j = q_i$

REMOVE any  $k \in G$

set  $u_j = k$

set  $q_k = q_k - (1 - s_j)$

put  $k$  in  $G$  if  $q_k > 1$  else put  $k$  in  $H$

} else {

REMOVE any  $k \in G$ , assign  $v_j = k$ ,  $s_j = 1$

}

To sample a value from this distribution

1. Choose cell index  $i \in [1, N]$  uniformly at random.
2. With probability  $s_i$  select value  $v_i$ , otherwise select  $u_i$ .

Example :  $p_2 = 0.4, p_6 = 0.1, p_8 = 0.2, p_9 = 0.3$ .

The table below illustrates the state of the algorithm after each step. The initial state is

value	probability	q	Set	index	v	u	s
2	0.4	1.6	$G$	0	-	-	-
6	0.1	0.4	$H$	1	-	-	-
8	0.2	0.8	$H$	2	-	-	-
9	0.3	1.2	$G$	3	-	-	-

---

For  $j = 1$  we choose  $2 \in G$  and  $6 \in H$ . Assign  $v_1 = 6, u_1 = 2, s_1 = 0.4$ , reassign  $q_2 = 1.6 - (1 - 0.4) = 1.0$ , return 2 to  $G$ :

value	probability	q	Set	index	v	u	s
2	0.4	1.0	$G$	1	6	2	0.4
6	0.1	—	—	2	-	-	-
8	0.2	0.8	$H$	3	-	-	-
9	0.3	1.2	$G$	4	-	-	-

---

For  $j = 2$  we choose  $8 \in H$  and  $9 \in G$ . Assign  $v_2 = 8, u_2 = 9, s_2 = 0.8$ , reassign  $q_9 = 1.2 - (1 - 0.8) = 1.0$ , return 9 to  $G$ :

value	probability	q	Set	index	v	u	s
2	0.4	1.0	$G$	1	6	2	0.4
6	0.1	—	—	2	8	9	0.8
8	0.2	—	—	3	-	-	-
9	0.3	1.0	$G$	4	-	-	-

---

$H$  is empty so we choose  $2 \in G$ , assign  $v_3 = 2, s_3 = 1.0$ .

$H$  is empty so we choose  $9 \in G$ , assign  $v_4 = 9, s_4 = 1.0$ .

value	probability	q	Set	index	v	u	s
2	0.4	—	—	1	6	2	0.4
6	0.1	—	—	2	8	9	0.8
8	0.2	--	—	3	2	-	1.0
9	0.3	—	—	4	9	-	1.0

Reconstructing the probabilities, if  $X$  samples from this distribution, then

- $X = 2$  if EITHER cell 1 is selected and then  $u_1$  is selected OR cell 3 selected and then  $v_3$  is selected. This happens with probability  $(1/4) * (1 - 0.4) + (1/4) * 1 = 0.4$ .
- $X = 6$  if cell 1 is selected and then  $v_1$  is selected. This happens with probability  $(1/4) * 0.4 = 0.1$ .
- $X = 8$  if cell 2 is selected and then  $v_2$  is selected. This happens with probability  $(1/4) * 0.8 = 0.2$ .
- $X = 9$  if EITHER cell 2 is selected and then  $u_2$  is selected OR cell 4 selected and then  $v_4$  is selected. This happens with probability  $(1/4) * (1 - 0.8) + (1/4) * 1 = 0.3$ .