Survivable IP Over WDM: An Efficient Mathematical Programming Problem Formulation

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Abstract: The problem of laying out a survivable IP network over a WDM network is considered. The links of the IP network are realized by unprotected lightpaths of the WDM network, and the lightpaths must be routed so that the IP network remains connected after any fault. Crochat and Le Boudec [2] introduced the problem for the case of single fiber-link failures, and Modiano and Narula-Tam [5] provided an integer linear programming problem (ILP) formulation. However, the ILP is difficult to solve. An alternative mixed integer linear programming (MILP) problem is provided that is easier to solve. In experiments, the run times of the MILP can be much faster. Variations of the MILP are also discussed that allow lightpaths to be protected, and can incorporate shared risk link groups.

1. Introduction

Wavelength division multiplexing (WDM) provides very wide communication bandwidth over optical fiber-links. It is essentially *frequency division multiplexing* of optical signals onto optical fibers, where *wavelengths* are equivalent to carrier frequencies (actually, the inverse of carrier frequencies). The technology has scaled well over time providing more wavelengths per fiber and higher bit rates per wavelength. This is important for the Internet, which is still experiencing exponential growth in traffic. WDM technology is providing some of the very high bandwidth links for broadband IP networks.

Figure 1 illustrates how an IP network may be overlaid on a WDM network. The WDM network is composed of *optical cross-connects* (OXCs) interconnected by fiber-links. The fiber-links are pairs of fibers, where fibers in a pair carry signals in opposite directions. Thus, the fiber-links and their wavelength channels are full duplex. An OXC cross-connects optical signals between its incident wavelength channels. The OXCs are wavelength converting so that the signals between any pair of channels may be cross-connected. The WDM network provide *lightpaths*, which are full duplex optical end-to-end connections. A lightpath starts and ends at OXCs, and is composed of a sequence of wavelength channels that are cross-connected at intermediate OXCs.

The IP network is composed of routers and links. The IP links are realized by lightpaths as shown in the figure. To facilitate this, the routers have their IP link ports directly connected to the OXCs. Since the IP network is carrying large amounts of data traffic, it is critical to provide protection against faults, such as fiber-link cuts.

There has been a considerable amount of effort to improve protection strategies for individual lightpaths (e.g., see [1]) but until recently there has been little effort to propose protection strategies for entire networks, and in particular IP networks. Crochat and Le Boudec [2] were perhaps the first to consider protecting an IP network over a WDM network. They recognized that an IP network is resilient to faults as long as it remains connected because its routing algorithm will adapt packet routes according to the current working topology. Of course, the convergence of the routes may take time but the delays may be tolerable to many IP users, who primarily use the Internet for web browsing, email, and downloading and uploading files. Crochat and Le Boudec proposed the following problem [2]. Given an IP network topology, find routes for the corresponding lightpaths so that the IP network remains connected (i.e., *survives*) after any single fiber-link fault. Note that the survivability requirement is very minimal, and the network bandwidth could drop considerably when a fault occurs. However, when compared to protecting every individual lightpath, network costs would be low which is important for cost sensitive Internet users. Although Crochat and LeBoudec proposed the problem, they did not describe it as a mathematical programming problem. They did not find an efficient algorithm that solved the problem exactly. Instead, they used a *tabu search* heuristic to find suboptimal solutions. Note that the problem is difficult to solve exactly, and has been shown to be NP-Complete [3][5].

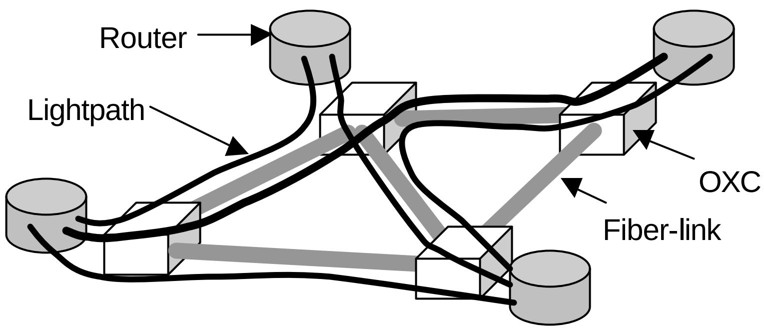

Figure 1. An IP network over a WDM network.

Recently, Modiano and Narula-Tam [5] found an *integer linear programming* (ILP) formulation of the problem. Now mathematical software packages can be applied to solve the problem exactly. A drawback of the formulation is that the number of its constraints grows exponentially with the size of the networks. This makes it difficult to solve for even moderate sized networks.

In Section 2, we will propose a modification of the ILP, and its number of constraints grows as a polynomial with the size of the networks. Our problem formulation is a *mixed integer linear programming* (MILP) problem formulation. We compare the ILP of [5] and our MILP by simulations. The MILP has much lower run times and in some cases by two orders of magnitude.

Note that for certain instances, the ILP and MILP will not have a solution, i.e., there exists no lightpath routing that will make the IP network survivable. In Section 3, we address this by proposing a variation of the problem, where each lightpath may be protected. Then a solution always exists because we could have all the IP links be realized by protected lightpaths. In the section, we also consider WDM networks that may have sets of links failing together. For example, fiber-links passing through a conduit will fail together if the conduit is cut. Such a set of fiber-links is referred as a *shared risk link group* (SRLG). Finally, we present our conclusions in Section 4.

2. Mathematical programming problem formulation

We will first review the ILP problem given in [5]. Then we will describe modifications that transform it into an MILP, which we refer to as MILP-1. Subsequently, we present experimental results to compare the difficulty in solving the formulations.

For the ILP problem of [5], a WDM network is represented by a graph (V_P, E_P), where V_P is the collection of OXCs and E_P is the collection of fiber-links (here, the subscript "P" corresponds to "physical" links). Let W denote the number of wavelengths carried by the fiber-links. There is an IP network represented by a graph (V_L, E_L), where V_L is the collection of IP routers and E_L is the collection of IP links (here, the subscript "L" corresponds to "logical" links). For simplicity, it is assumed that there is one router located at each OXC, and so the routers and OXCs are paired. We will use a set V to denote both V_P and V_L, and so a router-OXC pair is a *node* in V. We also assume V is an ordered set so that if $u, v \in V$ then either $u > v$ or $u < v$. Note that since the fiber-links are full duplex, if $(i, j) \in E_P$ then $(j, i) \in E_P$. Similarly, for IP links, if $(i, j) \in E_L$ then $(j, i) \in E_L$.

Each IP link is realized by a lightpath. The problem is to find routes for the lightpaths so that the IP network is survivable. In the ILP, the route of a lightpath for IP link $(s,t) \in E_L$ is represented by binary variables $\{f_{ij}^{st} : (i, j) \in E_P\}$. If IP link (s,t) is routed through fiber-link (i, j) then $f_{ij}^{st} = 1$, otherwise, $f_{ij}^{st} = 0$. We can also represent a route for the lightpath by another collection of variables $\{f_{ij}^{ts} : (i, j) \in E_P\}$. Since we only need one route, we will choose set $\{f_{ij}^{st} : (i, j) \in E_P\}$ if $s < t$, and choose set $\{f_{ij}^{ts} : (i, j) \in E_P\}$, otherwise.

If $\{f_{ij}^{st} : (i, j) \in E_P\}$ represents a valid route then it will correspond to a unit of "flow" from s to t. The ILP has flow conservation constraints to insure a proper flow: for each $i \in V$,

$$\sum_{(i,j)\in E_P} f_{ij}^{st} - \sum_{(j,i)\in E_P} f_{ji}^{st} = \begin{cases} 1 & \textit{if} \quad s = i \\ -1 & \textit{if} \quad t = i \\ 0 & \textit{otherwise.} \end{cases}$$

To guarantee that the lightpath routes lead to a survivable IP network the ILP has additional constraints. It uses the following definitions. A *cut-set* is a pair (S, $V \setminus S$), where S is a proper, nonempty subset of V, and $V \setminus S$ is the subset of nodes not in S. Note that a cut-set is a partition of V. An IP link (s,t) *crosses* the cut-set if $s \in S$ and $t \in V \setminus S$ or $t \in S$ and $s \in V \setminus S$. Here, (s,t) is an unordered pair, so (s,t) is the same as (t,s). Similarly, we say that a fiber-link (i,j) *crosses* the cut-set if $i \in S$ and $j \in V \setminus S$ or $j \in S$ and $i \in V \setminus S$. Let $CS(S, V\setminus S)$ denote the set of IP links (s,t) that cross the cut-set, and $|CS(S, V\setminus S)|$ denote their number. Figure 2 shows an example cut-set (S, $V\setminus S$), and in this case, $CS(S, V\setminus S) = 2$. The constraint is as follows: for each cut-set (S, $V\setminus S$) and each fiber-link $(i, j) \in E_P$ that crosses it,

$$\sum_{(s,t)\in CS(S,V\setminus S):s<t} \left(f_{ij}^{st} + f_{ji}^{st}\right) < \left|CS(S, V \setminus S)\right|.$$

The left hand side is an estimate of the number of lightpaths that "cross" the cut-set through fiber-link (i,j). Since the right hand side is the number of lightpaths that must cross the cut-set, the inequality insures that not all the lightpaths cross via the fiber-link (i,j). Otherwise, the failure of the fiber-link will disconnect S from $V \setminus S$.

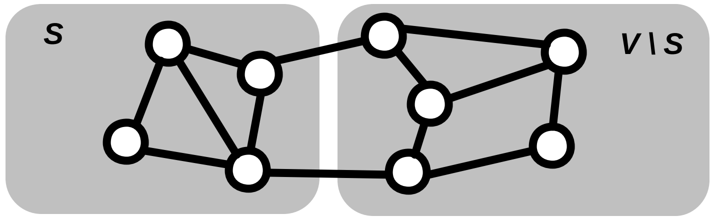

Figure 2. A cut set (S, $V \setminus S$).

Now we state the ILP. It has the binary variables $\left\{ f_{ij}^{st} : (i,j) \in E_P, (s,t) \in E_L, s < t \right\}$. Its objective is to minimize the number of wavelength channels used:

$$\text{minimize} \sum_{(i,j)\in E_P} \sum_{\substack{(s,t)\in E_L \\ s<t}} f_{ij}^{st},$$

subject to the following constraints.

C1. Integer flow constraints: For each $(i,j) \in E_P$ and $(s,t) \in E_L$ such that $s < t$, $f_{ij}^{st} \in \{0,1\}$.

C2. Capacity constraints: For each $(i,j) \in E_P$ such that $i < j$, $\sum_{\substack{(s,t)\in E_L: \\ s<t}} \left(f_{ij}^{st} + f_{ji}^{st} \right) \le W$. This allows a fiber-link (i,j) to carry at most W lightpaths. The restriction $i < j$, insures that there is only one inequality per fiber-link.

C3. Connectivity constraints (flow conservation): For each IP link $(s,t) \in E_L$, such that $s < t$, and each node $i \in V$,

$$\sum_{(i,j)\in E_P} f_{ij}^{st} - \sum_{(j,i)\in E_P} f_{ji}^{st} = \begin{cases} 1 & \text{if} \quad s = i \\ -1 & \text{if} \quad t = i \\ 0 & \text{otherwise}. \end{cases}$$

C4. Survivability constraints: For each cut-set (S, $V \setminus S$) and each fiber-link $(i,j) \in E_P$ that crosses it,

$$\sum_{(s,t)\in CS(S,V\setminus S):s<t} \left(f_{ij}^{st} + f_{ji}^{st} \right) < \left| CS(S, V \setminus S) \right|.$$

It was reported in [5] that the solving the ILP using a standard solver CPLEX required long run times. The culprit is the survivability constraints *C4*. The number of survivability constraints can be much larger than the number of proper cut-sets, which is $2^{|V|} - 2$. Thus, the number of the constraints grows exponentially with the number of nodes.

To lower the time complexity, one can approximate the ILP with another that is easier to solve. In [5], a *relaxed* version of the ILP is proposed that removes most of the inequalities of *C4*. The only ones left are for cut-sets (S, $V \setminus S$) where S is a single node. This insures that no node will have all its lightpaths leaving over a single fiber-link. It reduces run times considerably but may lead to invalid solutions as shown in Figure 3. Here, the lightpaths of each node leave through different fiber-links. However, the failure of fiber-link (3,4) disconnects IP links (3,5), (3,4), and (0,4), which disconnects the IP network.

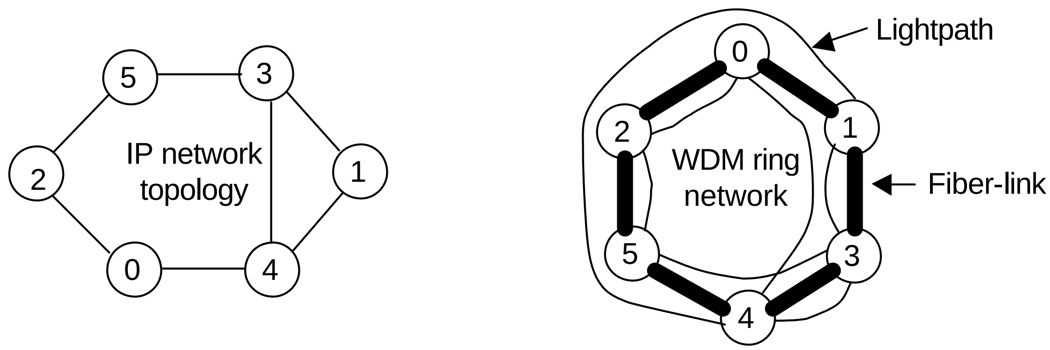

Figure 3. A vulnerable IP network lightpath layout over a WDM ring network.

Next we present our variation of the ILP which replaces the survivability constraints with a new set of constraints. The number of our new constraints grows as a polynomial function of the number of nodes, so it should be easier to solve. In addition, we introduce variables that do not have to be integer. Therefore, our problem is an MILP, which we refer to as MILP-1.

Before describing it, let us make some observations of the ILP of [5]. For each IP link $(s,t) \in E_L$ and fiber-link $(i,j) \in E_P$, let c_{st}^{ij} denote $1-(f_{ij}^{st}+f_{ji}^{st})$. Since the ILP minimizes the number of wavelength channels used, $f_{ij}^{st}+f_{ji}^{st}$ is at most 1. Also note that $f_{ij}^{st}+f_{ji}^{st}=1$ implies that the lightpath for (s,t) crosses fiber-link (i,j), and $f_{ij}^{st}+f_{ji}^{st}=0$ implies that the lightpath does not. Thus, c_{st}^{ij} indicates if the IP link (s,t) is *up* when fiber-link (i,j) fails. In particular, $c_{st}^{ij}=1$ implies the IP link is *up* and $c_{st}^{ij}=0$ implies it is *down*. Assuming each working IP link has unit capacity, we can interpret c_{st}^{ij} as the amount of surviving capacity for IP link (s,t) when fiber-link (i,j) fails.

Suppose fiber-link (i,j) fails. Then $\{c_{st}^{ij}:(s,t)\in E_L\}$ are the surviving capacities of the IP links. We impose constraints to insure the surviving IP network is connected. We require that the surviving IP network be able to carry a positive "flow" from each node to a particular node, say node 1. This implies all nodes are connected to node 1, and so the network is connected.

We need to choose the flow to be sufficiently small so that it will always be within the IP link capacity of 1. A flow of $\frac{1}{|V|-1}$ from each node, other than node 1, will work. Then node 1 will have to sink a total of 1 unit of flow. To see why this works, note that if the surviving IP network is connected then it has a spanning tree. The flows can be routed along the tree to node 1. Since a tree link will carry at most $|V|-1$ flows, its unit capacity is always sufficient to carry all its flows.

In MILP-1, for each fiber-link (i,j), we have new variables $\{r_{st}^{ij}:(s,t)\in E_L, s<t\}$. The variable r_{st}^{ij} is the amount of flow along IP link (s,t) when fiber-link (i,j) fails. It is constrained by $r_{st}^{ij}\le c_{st}^{ij}$. We replace the survivability constraints *C4* with the following constraints.

C'4 Survivability constraints. For each fiber-link (i,j) such that $i < j$, we have the following.

C'4.a. For each IP link $(s,t) \in E_L$ such that $s < t$,

$$0 \le r_{st}^{ij} \le 1 - (f_{ij}^{st} + f_{ji}^{st})$$
$$0 \le r_{ts}^{ij} \le 1 - (f_{ij}^{st} + f_{ji}^{st})$$

C'4.b. For each node $s \in V$,

$$\sum_{(s,t)\in E_L} r_{st}^{ij} - \sum_{(t,s)\in E_L} r_{ts}^{ij} = \begin{cases} -1 & \text{if} \quad s = 1 \\ \dfrac{1}{|V|-1} & \text{otherwise.} \end{cases}$$

The constraints *C'4* guarantee that if fiber-link (i,j) fails then the surviving links can support an amount $\dfrac{1}{|V|-1}$ of flow from all nodes to node 1. Therefore, the IP network will survive the fiber-link failure.

MILP-1 has twice as many variables as the ILP of [5] but a significantly smaller number of constraints, especially when $|V|$ is large. To compare the two formulations, we solved them using AMPL [4] and the popular CPLEX solver on a Sun Blade 100 workstation. AMPL is a modeling language for mathematical programming, and it has software that converts AMPL code to proper inputs for solvers such as CPLEX. We measured the *elapsed time* and *CPU time*. Elapsed time is the "wall clock time," while CPU time is the system plus user times.

Our WDM network and IP network topologies were generated randomly as follows. Initially, the network has no links and then links are added randomly until it is *2-connected*, i.e., the topology will remain connected over all possible single link deletions. Also, when a link is added, it must be between two nodes without a link between them, i.e., the resulting graphs do not have multiple links between a pair of nodes. We generated pairs of random topologies, where each pair corresponds to a particular number of nodes. For each pair, one topology represents an IP network and the other represents the WDM network. Each pair is an *instance* of a problem. We also assume *W* is very large, so we can ignore constraint *C2*.

Figure 4 shows the elapsed and CPU run times for the ILP of [5], MILP-1, and the relaxed version of the ILP, which are labeled, respectively, "ILP", "MILP", and "Relaxed-1". The instances correspond to the numbers of nodes 7 to 15. The run times are in seconds and are plotted on a log scale. We did not plot the CPU times for MILP-1 and Relaxed-1 because they are virtually identical to the elapsed times. For comparison, we also plot 1 minute and 1 hour. MILP-1 has much lower running times than ILP. For 15 nodes, MILP-1 has run times that are two orders of magnitude smaller than the ILP. Also note that the run times for the ILP does not scale well with the size of the networks. For 15 nodes, the elapsed time for the ILP is over an hour. In addition, observe that for the ILP, the elapsed and CPU times diverge at 13 nodes. At this point, the amount of memory required to solve the problem approaches the amount of available RAM. We suspect that the difference in run times may be due to virtual memory being used.

Figure 5 presents the elapsed and CPU run times for MILP-1 for larger numbers of nodes. MILP-1 scales reasonably well for large instances. The elapsed run times for 100 nodes are about an hour. Figure 5 also shows the run times for the relaxed ILP. It runs faster than

MILP-1 but, as mentioned earlier, it does not guarantee a proper solution. Again note that the elapsed and CPU times diverge at 70 nodes which we believe is due to virtual memory being used.

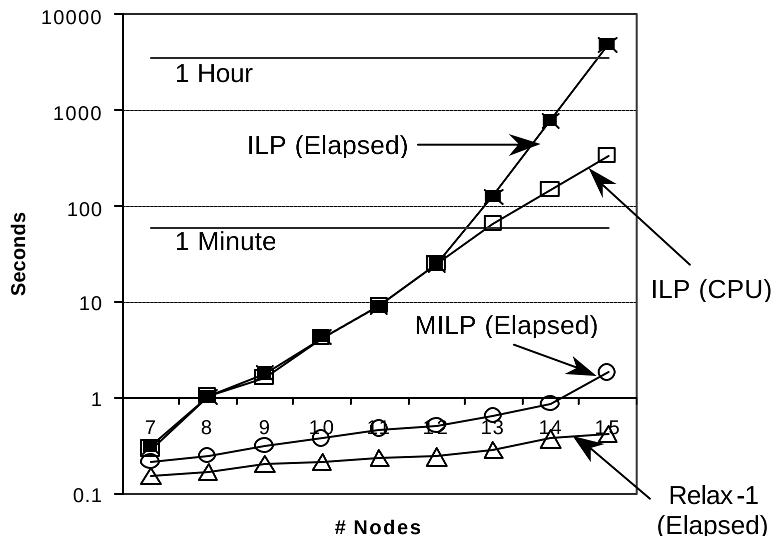

Figure 4. Run times for ILP, MILP-1, and the relaxed ILP.

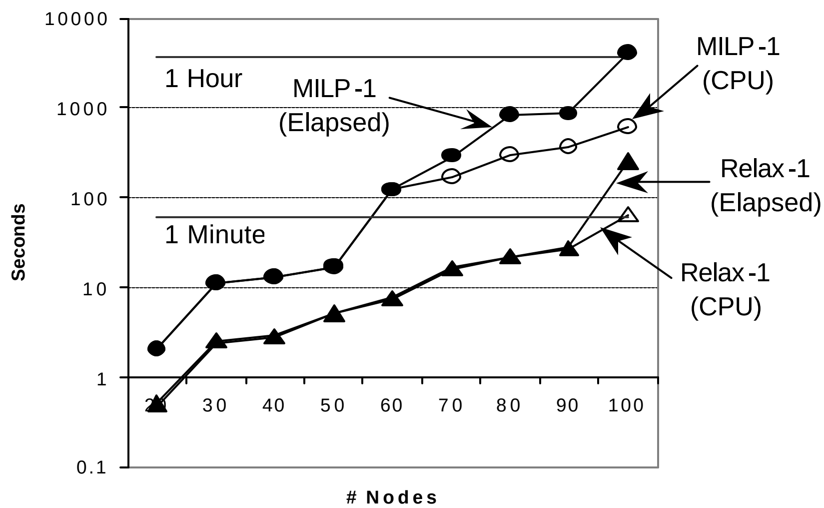

Figure 5. Run times for MILP-1 and the relaxed ILP.

3. Variations of the problem

We considered two variations of MILP-1, which we refer to as MILP-P and MILP-S. MILP-P allows lightpaths to be protected. Here, protection is dedicated, e.g., 1+1 or 1:1. MILP-S allows the WDM network to have SRLGs. We will discuss MILP-P and then MILP-S.

MILP-P is considered because there are instances of MILP-1 (and ILP) that have no solution. To check how typical this is we conducted experiments. The WDM network topologies we used were a 12 node ring and the two topologies shown in Figure 6. A hundred IP network topologies were randomly generated as described in Section 2. Note that all the IP network and WDM network topologies are 2-connected. For the 12 node ring, 3Cycle, and NSFNET, the percent of instances for MILP-1 without solution were, respectively, 21%, 8% and 1%. Note that the percent is higher for sparse topologies. Also note that 21% is quite high.

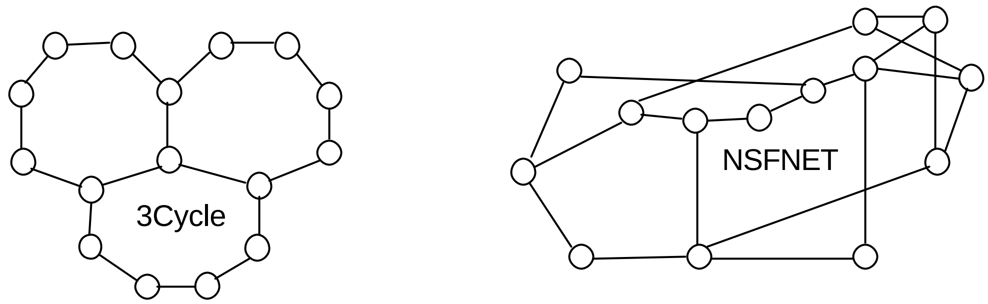

Figure 6. 3Cycle and NSFNET.

MILP-P allows a lightpath to be protected. If a lightpath is protected it has a *working* and a *protection* path, which in turn are basically two unprotected lightpaths. The working and protection paths are routed to avoid common fiber-links so that at least one will survive a fiber-link failure. It is assumed that the WDM network topology is 2-connected so that working and protection path pairs can be found between all pairs of nodes. If the IP network topology is connected then the MILP-P always has a solution because we can use protected lightpaths for all the IP links. However, an optimal solution avoids protected links because they have high cost.

The MILP-P is a modification of MILP-1 as follows. For each IP link $(s,t) \in E_L$ such that $s < t$, there is a new integer variable F^{st} which is restricted to have value of either 1 or 2. If the IP link (s,t) is realized by an unprotected lightpath then $F^{st} = 1$, and if it is realized by a protected lightpath then $F^{st} = 2$ (because there is a working and a protection path). The connectivity constraints *C3* are now replaced with

C"3.a. For each IP link $(s,t) \in E_L$ such that $s < t$, and each node $i \in V$,

$$\sum_{(i,j)\in E_P} f_{ij}^{st} - \sum_{(j,i)\in E_P} f_{ji}^{st} = \begin{cases} F_{st} & if \quad s = i \\ -F_{st} & if \quad t = i \\ 0 & otherwise. \end{cases}$$

This guarantees that if the IP link is realized by an unprotected lightpath then there is one unit of flow from s to t as before. On the other hand, if the lightpath is supposed to be protected then there are two units of flow, one per working and protection path. To insure that the working and protection paths avoid using the same fiber-links, we have the following additional constraints.

C"3.b. For each IP link $(s,t) \in E_L$ such that $s < t$, and $(i, j) \in E_P$ such that $i < j$, $f_{ij}^{st} + f_{ji}^{st} \le 1$.

In addition, constraints *C'4.a* of MILP-1 are replaced with the following constraints.

C"4.a. For each IP link $(s,t) \in E_L$ such that $s < t$,

$$0 \le r_{st}^{ij} \le F_{st} - (f_{ij}^{st} + f_{ji}^{st})$$
$$0 \le r_{ts}^{ij} \le F_{st} - (f_{ij}^{st} + f_{ji}^{st}).$$

Note that if the IP link is realized by an unprotected lightpath then $F^{st} = 1$ and constraints *C"4.a* are identical to the constraints *C'4.a* of MILP-1. If the IP link is realized by a protected lightpath then $F^{st} = 2$. Thus, $F_{st} - (f_{ij}^{st} + f_{ji}^{st}) \ge 1$ because $f_{ij}^{st} + f_{ji}^{st} \le 1$ (due to constraint *C"3.b*). Then r_{st}^{ij} and r_{ts}^{ij} can take values from 0 to 1. This is consistent with the fact that the IP link is protected and will survive all single fiber-link faults.

We ran AMPL and CPLEX on MILP-1 for the same IP network and WDM network topologies used in the simulations described in Section 2. Figure 7 shows the elapsed and CPU times for the topologies with 10, 20, ..., 100 nodes. Note that the run times are relatively close even though MILP-P has additional variables $\{F_{st}\}$.

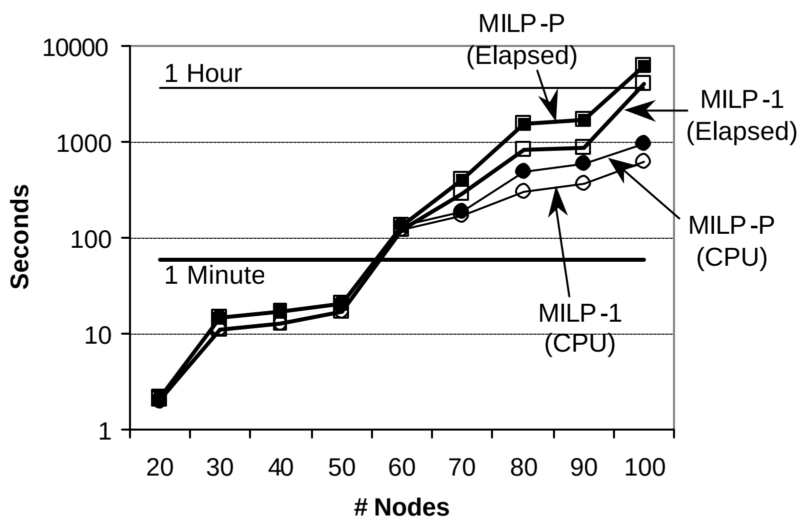

Figure 7. Run times for MILP-1 and MILP-P.

We now turn to discuss MILP-S. It is modified from MILP-1 using similar modifications as in MILP-P. Rather than give a detailed description of the modifications, we will present an outline. MILP-S allows unprotected and protected lightpaths and SRLGs. The possible fiber-link faults for the WDM network are m SRLGs which are denoted by $SRLG[k]$ for $k = 1, 2, ..., m$. Note that an SRLG only includes fiber-links (i,j) where $i < j$. In addition to each binary variable f_{ij}^{st} there is a binary variable p_{ij}^{st}. The variables f_{ij}^{st} define the working paths of the lightpaths, while the variables p_{ij}^{st} define the protection paths. Variables f_{ij}^{st} satisfy the connectivity constraints *C3*, while variables p_{ij}^{st} satisfy similar constraints shown below: for each $i \in V$,

$$\sum_{(i,j)\in E_P} p_{ij}^{st} - \sum_{(j,i)\in E_P} p_{ji}^{st} = \begin{cases} P_{st} & if \quad s = i \\ -P_{st} & if \quad t = i \\ 0 & otherwise. \end{cases}$$

where P_{st} is a binary variable that equals 1 if IP link (*s*,*t*) has a protection path, and equals 0 otherwise. To insure a working path for IP link (*s*,*t*) traverses a fiber-link (*i*,*j*) at most once, we have the constraint $f_{ij}^{st} + f_{ji}^{st} \le 1$, and similarly for protection paths, we have $p_{ij}^{st} + p_{ji}^{st} \le 1$.

Instead of having variables $\{r_{st}^{ij} : (s,t) \in E_P\}$ which correspond to a fault at fiber-link (*i*,*j*), MILP-S has variables corresponding to SRLGs, e.g., $\{r_{st}^{k} : (s,t) \in E_P\}$ corresponds to *SRLG*[*k*]. MILP-S has additional variables h_{st}^{k}, which is the amount of surviving bandwidth on the working path of IP link (*s*,*t*) when *SRLG*[*k*] is fails. Similarly, there are variables g_{st}^{k} for the protection paths. These variables must satisfy the constraints $h_{st}^{k} \le 1 - \left(f_{ij}^{st} + f_{ji}^{st}\right)$ and $g_{st}^{k} \le P_{st} - \left(p_{ij}^{st} + p_{ji}^{st}\right)$. The survivability constraints *C'4* are replaced with $r_{st}^{k} \le g_{st}^{k} + h_{st}^{k}$, $r_{ts}^{k} \le g_{st}^{k} + h_{st}^{k}$, and for each $s \in V$,

$$\sum_{(s,t)\in E_L} r_{st}^{k} - \sum_{(t,s)\in E_L} r_{ts}^{k} = \begin{cases} -1 & \textit{if} \quad s = 1 \\ \dfrac{1}{|V|-1} & \textit{otherwise.} \end{cases}$$

4. Conclusions

We provided an alternative to the ILP of [5] which has an exponential number of constraints. Our alternative is an MILP problem, we refer to as MILP-1, which has constraints that grow as a polynomial with the size of the networks. We verify by experiments that MILP-1 can be solved in much shorter time and in some cases by two orders of magnitude. We also described variations of MILP-1 that allowed protected lightpaths and SRLGs. These are practical extensions. Dedicated protection is the simplest and usually the first type of protection to be available. Also, SRLGs are multiple fiber-link failures that are practical design considerations.

Acknowledgements

This work was supported in part by Fujitsu Laboratories of America.

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